1 - Astronomical Tools

Purpose: To learn fundamental tools astronomers use on a daily basis. In your group, work the problems (Q1–Q14) on a separate sheet and submit one per person.
Due: next lab session, at the start of class.

Units:

All physical quantities have units: the length of one hair; the mass of an apple; the time it takes your coffee to cool down. Different countries use different units; in the USA, we persist in mostly using the English system. The fundamental units of the English system are the yard (yd), the pound (lb), and the second (s). Other common units of this system have strange multiples of the fundamental units:

1 ton = 2000 lb
1 ounce (oz) = 1/16 lb
1 mile (mi) = 1760 yd
1 inch (in) = 1/36 yd

Astronomy and most of the world have adopted the metric system, the *Systeme International d'Units* (SI), with the following fundamental units:

- meter (m) for length
- kilogram (kg) for mass
- second (s) for time

The system of units based on the meter, kilogram, and second is also known as the mks system. The system of units based on the centimeter, gram, and second is known as the cgs system. All other units in the SI system are based on multiples of 10, and their names change only in their prefixes (see Table 1).

Table 1. Metric Prefixes

Prefix	Abbreviation	Value
deci-	d	10^{-1}
centi-	c	10^{-2}
milli-	m	10^{-3}
micro-	μ	10^{-6}
nano-	n	10^{-9}
pico-	p	10^{-12}
femto-	f	10^{-15}
atto-	a	10^{-18}

Prefix	Abbreviation	Value
deca-	da	10^{1}
hecto-	h	10^{2}
kilo-	k	10^{3}
mega-	M	10^{6}
giga-	G	10^{9}
tera-	T	10^{12}
peta-	P	10^{15}
exa-	E	10^{18}

Converting Units:

In the USA, we must convert between English and metric units because all science and international commerce is transacted in metric units. Fortunately, converting units is not difficult. Most of the exercises in this lab will use metric units. Use Tables 2 and 3 to convert to quantities you are used to on a daily basis to those we will use in this course:

Table 2. English to Metric

English		Metric
1 in	=	2.54 cm
1 mi	=	1.609 km
1 lb	=	0.4536 kg†
1 gal	=	3.785 L

Table 3. Metric to English

Metric		English
1 m	=	39.37 in
1 km	=	0.6214 mi
1 kg	=	2.205 lb
1 L	=	0.2642 gal

† Strictly speaking kilograms (kg) are a measure of mass and pounds (lb) are a measure of weight (or force of gravity). Therefore, the conversion of lb to kg is only valid on Earth. The unit of weight/force in the metric system is the newton (N) and the unit of mass in the English system is the slug.

Many people have trouble converting between units and they don't know if they need to multiply or divide by the conversion factor. We offer a simple method to handle this problem.

Fact: any number multiplied by 1 equals itself. We also know that 1 divided by 1 is 1. We can use these simple properties to work out the correct value and unit of certain physical property, e.g., the mass of an object.

Suppose we wish to know how many kilograms a 150-kg person weighs. From the conversion tables, we know that 1 kg = 2.205 lb. Thus, we can construct the number 1 in two ways:

$$1 = \frac{1\ \text{kg}}{2.205\ \text{lb}} \qquad \text{or its reciprocal} \qquad 1 = \frac{2.205\ \text{lb}}{1\ \text{kg}}$$

Note that the 1's are dimensionless. In other words, the quantity in the numerator (top of fraction) is exactly equal to the number in the denominator (bottom). If we did not include the units, then we would be incorrectly saying that 1 = 1/2.205 *and* 1 = 2.205, which is nonsense. We need to keep the units!

To convert 150 lb into kilograms, we will multiply 150 lb by 1. This will not change the value of the "mass." However, how do we choose the correct 1? The answer is, we choose the 1 that will cancel the units we are trying to convert (lb in our case) while keeping the one we want to express our quantity in (kg in our case):

Useful:

$$
\begin{aligned}
150\ \text{lb} \times 1 = 150\ \text{lb} \times \frac{1\ \text{kg}}{2.205\ \text{lb}} \\
&= 150 \times \frac{1\ \text{kg}}{2.205} \\
&= \frac{150\ \text{kg}}{2.205} \\
&= 60.03\ \text{kg} \\
&\rightarrow 150\ \text{lb} = 60.03\ \text{kg}
\end{aligned}
$$

Useless:

$$
\begin{aligned}
150\ \text{lb} \times 1 = 150\ \text{lb} \times \frac{2.205\ \text{lb}}{1\ \text{kg}} \\
&= \frac{150 \times 2.205\ \text{lb}^2}{1\ \text{kg}} \\
&= 330.75\ \frac{\text{lb}^2}{\text{kg}} \\
&\rightarrow 150\ \text{lb} = 330.75\ \frac{\text{lb}^2}{\text{kg}}
\end{aligned}
$$

Note: Units can be "mixed" and be written in different ways. The following are all equivalent: $50\ \text{mph} = 50\ \text{mi/h} = 50\frac{\text{mi}}{\text{h}} = 50\ \text{mi h}^{-1}$.

Questions:

On your separate sheet, write your name and partner's name. Show your work for the following conversions (especially what happens to the units):

Q1) 25 miles (mi) into kilometers (km)

Q2) 50 miles per hour (mph) into kilometers per hour (km/h)

Q3) 60 kilograms (kg) into pounds (lb)

Q4) 9.81 meters per second per second (m/s^2) into centimeter per second-squared (cm/s^2)

Scientific Notation:

Astronomers deal with microscopic and macroscopic quantities. The age of the Universe is 15,000,000,000 years. The distance to the Sun is 149,600,000,000 meters. It is inconvenient to write them with so many zeros. Instead and to simplify, scientists use the power-of-ten notation. What this means is that numbers are expressed with an exponent that tells you how many times to multiply by ten.

For example:

$$10 = 10^1$$
$$100 = 10^2$$
$$0.01 = 10^{-2}$$
$$1 = 10^0$$

Using this method, we can write the age of the Universe as 1.5×10^{10} yr and the distance to the Sun as 1.496×10^{11} m. A number in scientific notation is written as

$$a\times10^n,$$

where a is called the coefficient and is a number greater than 1 and less than 10, and n is the exponent and is an integer. Some scientific calculators and programming languages use E or EE for the "×10" part (i.e., the Sun is 1.496E11 m away). Familiarity with scientific notation helps most in Lab 5: Seasons.

Questions:

On your separate sheet, express the following quantities (which have units) in scientific notation:

Q5) The U.S. federal debt of $7,000,000,000,000

Q6) 3040 s

Q7) 0.00012 m

Significant Figures:

While working on the previous practice problems, you should ask yourself, how many numbers should I keep after the period? Numbers should be given only to the accuracy that they are known with certainty or to the extent that they are relevant. For example, your weight may be measured to be 125 lb when in fact at that instant it was 124.7326 lb.

Question:

On your separate sheet, answer the following give the above discussion:

> **Q8)** More than a rounding problem, why is it irrelevant to state your weight with so many numbers, i.e. with so much precision?

In this lab, you will be measuring a number of quantities, and we will ask you to provide answers that involve the precision and accuracy. In experimental sciences, precision and accuracy have different meanings.

<u>Precision:</u> When you measure a quantity many times and you get very similar values, your measurement can be said to be precise.
<u>Accuracy:</u> When you measure a quantity and it is close to the true value, your measurement can be said to be accurate.

We can find examples when experiments are precise but not accurate, accurate but not precise, not precise and not accurate but the ones that are of value are those that are both accurate and precise.

<u>Example of precision:</u> Let's say you measure the length of the hallway with a meter stick. It is 25.2031498 meters. This measurement is too precise, since you would not be able to measure this value with a meter stick. You would need a laser. As a rule of thumb, the precision of a measurement is set by 1/2 or a 1/4 of the smallest division in your equipment.

<u>Example of accuracy:</u> Let's say you measure the temperature of your body many times, and they are: 55.2°F, 55.3°F, 55.4°F, 55.2°F, 55.4°F. These measurements are precise (all about the same) but not accurate since your body should have a temperature of about 98.6°F. It seems like the thermometer is not properly calibrated.

The precision and accuracy of our experiment will dictate the number of significant digits. In the example of precision, we should have expressed the length of the hallway with 3 significant digits: 2.52×10^{1} m or 2.52×10^{-2} km. The same number with 2 significant digits would be 2.5×10^{1} m.

Math Review:

<u>Circle of radius R:</u>

- the circumference or perimeter of a circle is $2\pi R$
- the area is πR^2

<u>Sphere of radius R:</u>

- the surface are of a sphere is $4\pi R^2$
- the volume is $4\pi R^3/3$.

<u>Angles:</u>

- there are 360 degrees in a full circle. The shorthand for degree is °.
- there are 60 minutes of arc in one degree. The shorthand for minute of arc or arcminute is ′ so 6 arcmin = 6′. There are $360^\circ \times 60'/^\circ = 21{,}600'$ in a full circle.
- there are 60 seconds of arc in one arcminute. The shorthand for second of arc or arcsecond is ″ so 7 arcsec = 7″. There are $360^\circ \cdot \frac{60'}{1^\circ} \cdot \frac{60''}{1'} = 1{,}296{,}000''$ in a full circle.
- angles can also be expressed in radians. The shorthand for radians is rad. There are 2π rad = 6.283185… rad in a full circle.

$$2\pi \text{ rad} = 360^\circ \rightarrow 1 \text{ rad} = \frac{360^\circ}{2\pi} = 57.3^\circ$$

<u>Conversions of angles:</u>

$$360^\circ = 2\pi \text{ rad} \rightarrow 1^\circ = \frac{2\pi}{360^\circ} = 0.01745 \text{ rad}$$

Questions:

On your separate piece of paper, answer the following and show your work.

Q9) How many degrees are there in 1/4 of a circle?

Q10) How many radians are there in 1/2 of a circle?

Q11) How many arcminutes (′) are there in 3″?

Trigonometric Functions:

Throughout this course we will occasionally use trigonometric (trig) functions: sine, cosine, and tangent with the help of special triangles.

In any right-angle triangle, where one angle is 90 degrees, the longest side is called the hypotenuse (hyp). This side is the one opposite of the right angle. The side of the triangle touching the angle you are studying (α in Figure 1) is called the adjacent (adj) and the side opposite the angle is called opposite (opp).

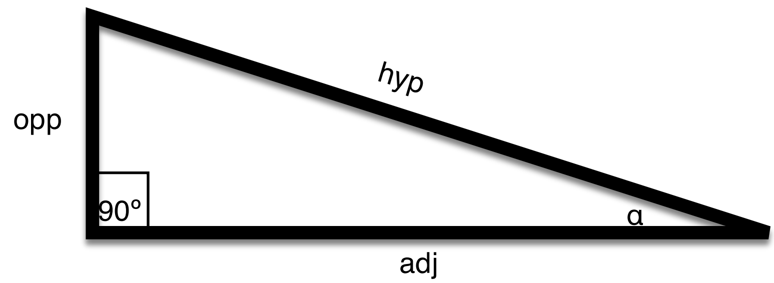

Figure 1: Right angle triangle.

$$\sin\alpha = \frac{\text{opp}}{\text{hyp}} \qquad \cos\alpha = \frac{\text{adj}}{\text{hyp}} \qquad \tan\alpha = \frac{\text{opp}}{\text{adj}}$$

From Pythagoras, we know that:

$$\text{hyp}^2 = \text{opp}^2 + \text{adj}^2$$

Small-Angle Approximation:

In astronomy, most of the angles we measure are extremely small. For example, try drawing a right triangle with an angle $\alpha = 1'$. It should be very difficult indeed. When you try to do this, could you tell the difference in length between the adjacent length and the hypotenuse? In fact, for $\alpha = 1'$, the difference between the hypotenuse and the adjacent is less than 0.02%. What this means is that whenever we work with small angles we can use a simplified trigonometric formula:

(size or height of object) ≈ (distance to object) × (angular size of object in radians)

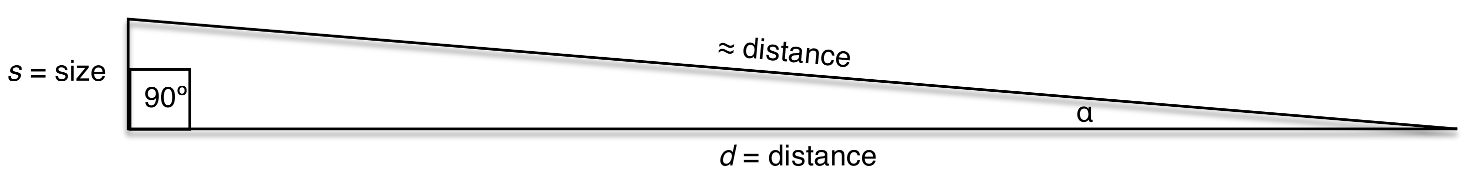

Figure 2: Right triangle for small angle α.

$$s \approx d \times \alpha \text{ (radians)}$$

Beware that the angle has to be in radians, and the height or size of the object and the distance to the object will be in the same physical units (both kilometers, both miles, both parsec, etc.).

Questions:

On your separate sheet of paper, answer the following. Show your work.

Q12) Using a ruler, draw a tall, skinny, vertical, roughly equilateral triangle a blank sheet of paper.

Draw a horizontal line (parallel to the base) at three evenly spaced heights in the triangle.

Carefully measure and label the distance, d, from the apex (top angle, α) to each horizontal line and the base.

In the same units, measure and label the width or size, s, of the horizontal lines (including the base).

Calculate and label the ratio of the horizontal size to the vertical distance ($s \div d$). These ratios are all roughly the same because the apex angle α is small and shared for all similar triangles you drew. We use this property in Lab 4: Distances and Sizes and Lab 7: Pinhole & Telescope.

Q13) Imagine you face STB. If you stand with your nose up against the building, it will occupy your entire view. As you back away, the building appears to get smaller; it covers a smaller and smaller angle (i.e., angular size), even though the size of the building has not changed at all. You can see that (i) your distance to the STB, (ii) the actual size of the building, and (iii) the angular size all work together so that you only need to know two of them to determine the third. Imagine that the STB subtends an angle of 1 degree. You know its height is 20 m. How far are you from the building? Draw a triangle and make the correspondence between the generic angles in Figure 2 with those relevant to this problem.

Q14) Imagine you see a building far away with an angular size of 1 degree. If you are 10 km from the building, how tall is the building? Calculate the lengths of the hypotenuse and the opposite side. Include a figure with all sides and angles labeled.