4 - Estimating Size and Distances

Purpose: To estimate the distance to an object in terms of its size and to estimate the size of an object in terms of its distance, using equipment always available: your body. To receive full credit, (i) answer all 27 questions (Q1–Q27), on a separate piece of paper with your and your partners' names and (ii) show your work. Some work takes place outside of lab.
Due: next week, at the start of class.

Materials:

1 one-dollar bill	string
1- & 2-meter sticks	tape
measuring tape	calculators

Pre-lab Questions—What do you think?:

Write your best guess answers to Q1 and Q2 on your sheet of paper.

Q1) If you hold out your hand at arm's length, what part or parts of your hand or fingers will just cover the (full) Moon?

Q2) How many Moon diameters fit into the distance between Earth and the Moon?

Procedure:

1. Tape a fake-dollar bill to a wall with the short side of the bill parallel to the floor.

2. Stretch an arm out to full length and make a fist, palm up. Straighten your little finger, keeping the rest of your hand closed in a fist. Position your hand so that your little finger is straight up and down. With your arm stretched in front of you, close one eye (see Figure 1).

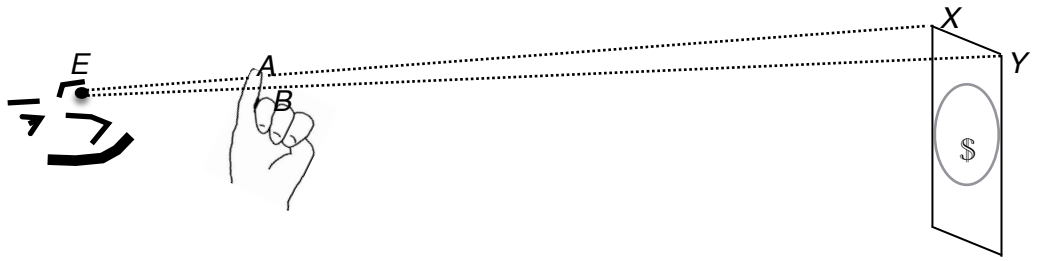

Figure 1: *Triangle formed by your eye (E), your finger width (AB), and the "dollar" bill's width (XY). The line AB is the width of your little finger stretched at arm's length and line XY is the width of the "dollar" bill taped to the wall. The distance from your eye to your finger is EA or EB, and the distance from your eye to the dollar bill is EX or EY.*

3. Stand close to the "dollar" bill where your little finger is not covering the entire bill and walk away from the bill until you see that the upper joint of your little finger appears to exactly cover the width of the dollar bill. Don't move.

4. Let your lab partner(s) measure the distance from your finger to your eye. Keep your arm straight and horizontal. Be careful not to poke each other's eyes. Don't move.

   Q3) On your separate piece of paper, record the distance from your finger to your eye in centimeters (also label what this value is!) Note: 1 in = 2.54 cm.

   Let your lab partner(s) measure the distance from your eye to the "dollar" bill. You can now move.

   Q4) Record and label the distance from your eye to the bill in centimeters.

5. Measure the width of the upper joint of your little finger.

   Q5) Record and label the width of your little finger in centimeters.

   Q6) Calculate the number of "finger-widths" that fit into your "eye-to-finger distance": $\frac{\text{Eye-to-Finger Distance}}{\text{Finger Width}} =$ …
   Explicitly show what happens to the units.
   Answer to Q6 is the number of finger-widths that will fit into the distance between your eye and your finger.

7. Measure the width of the "dollar" bill.

   Q7) Record and label the width of the "dollar" bill in centimeters.

   Q8) Compute how many "dollar-bill-widths" fit between your eye and the "dollar" bill: $\frac{\text{Eye-to-Bill Distance}}{\text{Dollar Bill Width}} =$ …
   Answer to Q8 is the number of "dollar"-bill-widths that fit into the distance between you and the "dollar" bill when you are standing at a position where your outstretched little finger just covers the "dollar" bill (i.e., Q4).

8. Repeat the procedure (steps 1–7) so your partner(s) can determine her/his own "eye-to-finger-width" relationship.

   Q9) The expectation is that your Q6 ratio should roughly equal your Q8 ratio. Explain why.

Q10) Calculate your percent error: $\%\ \text{Error} = \frac{|Q6 - Q8|}{\frac{1}{2}(Q6 + Q8)} \times 100\%$

You should have less than a 10% error. Talk to the instructor or TA if this isn't the case.

Q11) Discuss how your partner's Q6 and Q8 ratios compare to yours and what that means about how far they were standing compared to you and about human anatomy.

The Math:

The triangles *AEB* and *XEY* are known as *similar triangles*, i.e., the angles in triangle *AEB* are all the same as those in *XEY* (see Figure 2). This equality also means: the ratios of the side-to-base of these triangles are equal (e.g., *EA*/*AB* = *EX*/*XY*) *and* the ratios of the height-to-base of the triangles are the same:

$$\frac{\text{Distance from eye to finger}}{\text{Finger Width}} = \frac{\text{Distance from eye to dollar bill}}{\text{Dollar-Bill Width}}$$

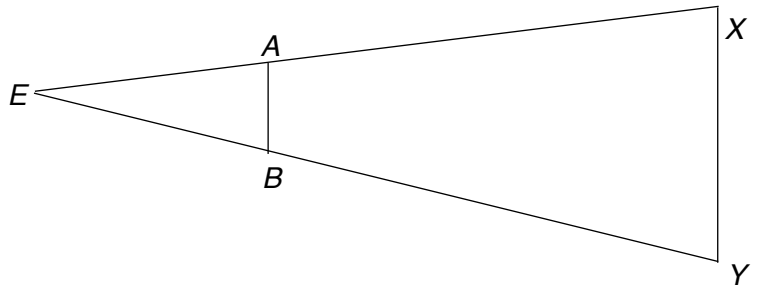

Figure 2: As seen from above, Figure 1 contains two similar triangles, as shown here. One is formed by your eye (E) and your finger width (AB), and another by your eye and the "dollar"-bill's width (XY).

9. Now stand at a position where the "dollar" bill is just exactly covered by two little finger-widths. This is illustrated by the following ratio:

$$\frac{\text{Distance from eye to finger}}{\text{2 Finger Width}} = \frac{\text{Distance from eye to dollar bill}}{\text{Dollar-Bill Width}}$$

Q12) Has the apparent size of the "dollar" bill increased, decreased, or stayed the same?

Q13) What has happened to the distance between you and the "dollar" bill?

Q14) Measure and record the distance between you and the "dollar" bill in cm.

Q15) Compare this distance to that when your little finger covered the "dollar" bill in Q4 by calculating the ratio between the distance in Q4 to Q14. How many times are you closer or farther from the "dollar" bill when you cover it with two fingers instead of one? Explain whether this makes sense and why or why not.

10. Move to a position where the "dollar "bill is just covered by only half of your finger width. This comparison is illustrated by the following ratio:

$$\frac{\text{Distance from eye to finger}}{\frac{1}{2}\text{ Finger Width}} = \frac{\text{Distance from eye to dollar bill}}{\text{Dollar-Bill Width}}$$

Q16) Has the apparent size of the "dollar" bill increased, decreased, or stayed the same?

Q17) What has happened to the distance between you and the "dollar" bill?

Q18) Measure and record the distance between you and the "dollar" bill in cm.

Q19) Compare this distance to that when your little finger covered the "dollar" bill in Q4 by calculating the ratio between the distance in Q4 to Q18. How many times are you closer or farther from the "dollar" bill when you cover it with 1/2 fingers instead of one? Explain whether this makes sense and why or why not.

11. Field Work A (during lab): Go out of the classroom and look for a large object far away. Find a location where you can just exactly cover the object's width with your little finger when your arm is fully extended.

Q20) Using what you have learned so far, determine how many object-widths will fit between you and the object. Explain your reasoning (hint: see Q9). This must be consistent with your Q6 and/or Q8 data.

Q21) Describe the object you chose and make an educated guess of its width (include the unit of measurement, i.e. cm, feet, etc.). Record and label this value.

Q22) Using the same method you used in Q20 and your guess in Q21, calculate the distance to the object (include unit of measurement). Show all your work. Explain whether this makes sense and why or why not.

12. Field Work B (complete Q21 in-lab and Q22–25 during the week, with or without your partner(s)):

Q23) Predict where you think the Moon will be in your local sky now, given that the last new Moon was Mon February 8. Discuss with your lab partner, consult with the instructor and TA. Explain your thought process and record what you expect for when you actually go and observe:

- Lunar phase
- Time
- Date
- Approximate cardinal direction of the Moon
- Approximate altitude of the Moon

Q24) As you go out (now or during the week), orient yourself by finding the cardinal directions and altitude you predicted in Q23. Discuss how close or far the Moon was from where you thought it would be and any other differences in your predictions and observations.

Use your little finger with your arm outstretched to measure the full apparent diameter of the Moon.

Q25) How many finger widths or fractions thereof just cover the Moon? Even if the Moon is a crescent, you can estimate its full diameter by measuring from one tip of the crescent across to the other tip.

Q26) State how many Moon-diameters the Moon is away from you. This must be consistent with your Q9 and Q18 answers.

Q27) Discuss how the answers to Q25 and Q26 compare with your predictions in Q1 and Q2.