1 - Astronomical Tools

Purpose: To learn fundamental tools astronomers use on a daily basis. Turn in all 13 problems on a separate sheet. Due in one week at the start of class.

Units

All physical quantities have units. The length of one of your hairs, the mass of an apple, the time it takes your coffee to cool down. Different countries use different units, alas in the USA we continue to use the English system. The fundamental units of the English system are the yard (**yd**), the pounds (**lb**), and the second (**s**). Other common units of this system have strange multiples of the fundamental units:

1 ton = 2000 lbs   1 ounce = 1/16 lb
1 mile = 1760 yd   1 inch = 1/36 yd

Astronomy and most of the world have adopted the metric system, the ***S**ysteme* ***I**nternational d'Units* (SI), with the following fundamental units:

- meter (**m**) for length
- kilogram (**kg**) for mass
- second (**s**) for time

The system of units based on the **m**eter, **k**ilogram, and **s**econd is also known as the **mks** system. The system of units based on the **c**entimeter, **g**ram, and **s**econd is known as the **cgs** system. All other units in the SI system are based on multiples of 10 and their names change only in their prefixes (see Table 1.).

Prefix	Abbreviation	Value
deci-	d	10^{-1}
centi-	c	10^{-2}
milli-	m	10^{-3}
micro-	μ	10^{-6}
nano-	n	10^{-9}
pico-	p	10^{-12}
femto-	f	10^{-15}
atto-	a	10^{-18}

Prefix	Abbreviation	Value
deca-	da	10^{1}
hecto-	h	10^{2}
kilo-	k	10^{3}
mega-	M	10^{6}
giga-	G	10^{9}
tera-	T	10^{12}

Table 1. Metric Prefixes

Conversions

In the USA, we must convert between English and metric units because all science and international commerce is transacted in metric units. Fortunately, converting units is not difficult. Most of the exercises in this lab will use metric units. Use Tables 2 and 3 to convert to quantities you are used to on a daily basis to those we will use in this course:

Table 2. English to Metric

English		Metric
1 inch	=	2.54 cm
1 mile	=	1.609 km
1 lb	=	0.4536 kg†
1 gal	=	3.785 liters

Table 3. Metric to English

Metric		English
1 m	=	39.37 inches
1 km	=	0.6214 mile
1 kg	=	2.205 lb
1 liter	=	0.2642 gal

† Strictly speaking kilograms are a measure of mass and pounds (lb) are a measure of weight. Therefore, the conversion of lb to kg is only valid on Earth. The unit of weight in the metric system is the Newton (N) and the units of mass in the English system is the slug.

Conversion of Units

Many people have trouble converting between units and they don't know if they need to multiply of divide by the conversion factor. We offer a simple method to handle this problem.

When multiplying any number by 1, the value equals itself. We also know that 1 divided by 1 is 1. We can use these simple properties to work out the correct value and unit of certain physical property, e.g. the mass of an object.

Suppose we wish to know how many kilograms a 150-lb a person weighs. From the conversion tables, we know that 1kg = 2.205 lb. Thus, we can construct the number 1 in two ways:

$$1 = \frac{1\ \text{kg}}{2.205\ \text{lb}}$$

or its reciprocal

$$1 = \frac{2.205\ \text{lb}}{1\ \text{kg}}$$

Note that the 1's are dimensionless, i.e. they do not have any units. In other words, the quantity in the numerator is exactly equal to the number in the denominator. If we did not include the units, then we would be incorrectly saying that 1=2.205 which is nonsense. We need to keep the units!
To convert 150-lb into kilograms, we will multiply 150-lb by 1. This will not change the value of the "mass." However, how do we choose the correct 1? The answer is, we choose the 1 that will cancel the units we are trying to convert (lb in our case) while keeping the one we want to express our quantity in (kg in our case):

Useful:

$$\begin{aligned} 150 \text{ lb} \times 1 &= 150 \text{ lb} \times \frac{1 \text{ kg}}{2.205 \text{ lb}} \quad \text{(cancel the two units of lb)} \\ &= 150 \times \frac{1 \text{ kg}}{2.205} \\ &= \frac{150 \text{ kg}}{2.205} \\ &= 68.03 \text{ kg} \\ &\rightarrow 150 \text{ lb} = 68.03 \text{ kg} \end{aligned}$$

Useless:

$$\begin{aligned} 150 \text{ lb} \times 1 &= 150 \text{ lb} \times \frac{2.205 \text{ lb}}{1 \text{ kg}} \quad \text{(can not cancel any unit)} \\ &= \frac{150 \times 2.205 \text{ lb}^2}{1 \text{ kg}} \\ &= 330.75 \ \frac{\text{lb}^2}{\text{kg}} \\ &\rightarrow 150 \text{ lb} = 330.75 \ \frac{\text{lb}^2}{\text{kg}} \end{aligned}$$

Problem: On a separate sheet, write your name and work out the following conversions:

1) 25 miles into kilometers
2) 50 miles per hour into kilometers per hour
3) 60 kg into lb
4) 9.8 m/s^2 into cm/s^2

Scientific Notation

Astronomers deal with microscopic and macroscopic quantities. The age of the Universe is 15,000,000,000 years. The distance to the Sun is 149,600,000,000 meters. It is inconvenient to write them always like that. Instead and to simplify, scientists use the power-of-ten notation. What this means is that numbers are expressed with an exponent that tells you how many times to multiply by ten. For example:

$$10 = 10^1$$
$$100 = 10^2$$
$$0.01 = 10^{-2}$$
$$1 = 10^0$$

Using this method, we can write the age of the Universe as 1.5×10^{10} years and the distance to the Sun as 1.496×10^{11} meters. A number in scientific notation is written as

$\mathbf{a} \times 10^{\mathbf{n}}$,

where **a** is called the coefficient and is a number less than 10, and **n** is the exponent and is an integer.

Problem: On a separate sheet, write your name and express the following numbers in scientific notation:

5) The U.S. federal debt of $7,000,000,000,000
6) 3040
7) 0.00012
8) While working on the previous practice problems, you should ask yourself, how many numbers should I keep after the period. Numbers should be given only to the accuracy that they are known with certainty or to the extent that they are relevant. For example, your weight may be measured to be 125 lb when in fact at that instant it was 124.7326 lb. More than a rounding problem, why is it irrelevant to state your weight with so many numbers, i.e. with so much precision?

In this lab, you will be measuring a number of quantities and we will ask you to provide answers that involve the precision and accuracy. In experimental sciences, precision and accuracy have different meanings.

Precision:
When you measure a quantity many times and you get very similar values, your measurement can be said to be precise.

Accuracy:
When you measure a quantity and it is close to the true value, your measurement can be said to be accurate.

We can find examples when experiments are precise but not accurate, accurate but not precise, not precise and not accurate but the ones that are of value are those that are both accurate and precise.

Example of precision: Let's say you measure with a meter stick the length of the hallway to be 25.2031498 meters. This measurement is too precise and you would not be able to measure it with a meter stick. You would need a laser. As a rule of thumb, the precision of a measurement is set by half or a quarter of the smaller division in your equipment.
Example of accuracy: Let's say you measure the temperature of your body many times and they are: 55.22°F, 55.32°F, 55.41°F, 55.21°F, 55.43°F. These measurements are precise but not accurate as your body should have a temperature of about 98°F.

The precision and accuracy of our experiment will dictate the number of significant digits. In the example of precision if the precision is 0.3m, we should have expressed the length of the hallway with 3 significant digits: 2.50×10^{1}m or 2.50×10^{-2} km. The same number with 2 significant digits would be 2.5×10^{1} m.

Math Review:

Circle of radius R:

- the circumference or perimeter of a circle is $2\pi R$
- the area is πR^2

Sphere of radius R:

- the surface of a sphere is $4\pi R^2$
- the volume is $4\pi R^3/3$.

Angles:

- there are 360 degrees in a full circle. The shorthand for degree is °.
- there are 60 minutes of arc in one degree. The shorthand for minute of arc or arc minute is ' so a 6 arcminutes = 6'. There are 360°x60'/° = 21,600' in a full circle.
- there are 60 seconds of arc in one arcminute. The shorthand for second of arc or arc second is "so a 7 arc sec = 7". There are 360°x60'/° x60'/"= 1.296,000" in a full circle.
- angles can also be expressed in radians. The shorthand for radians is rad. There are 2π rad = 6.283185… rad in a full circle.

Conversions of angles:

$$2\pi \text{ rad} = 360^\circ \rightarrow 1 \text{ rad} = \frac{360^\circ}{2\pi} = 57.3^\circ$$

$$360^\circ = 2\pi \text{ rad} \rightarrow 1^\circ = \frac{2\pi}{360^\circ} = 0.01745 \text{ rad}$$

Practice:

9) How many degrees are there in 1/4 of a circle?
10) How many radians are there in 1/2 a circle?
11) How many arc minutes are there in 3" (arcseconds)?

Trigonometric functions

Throughout this course we will occasionally use trigonometric (trig) functions: sine, cosine, and tangent with the help of special triangles.

In any right angle triangle, where one angle is 90 degrees, the longest side is called the hypothenuse (hyp). This side is the one opposite of the right angle. The side of the triangle touching the angle you are studying (α in Figure 1) is called the adjacent (adj) and the side opposite the angle is called opposite (opp).

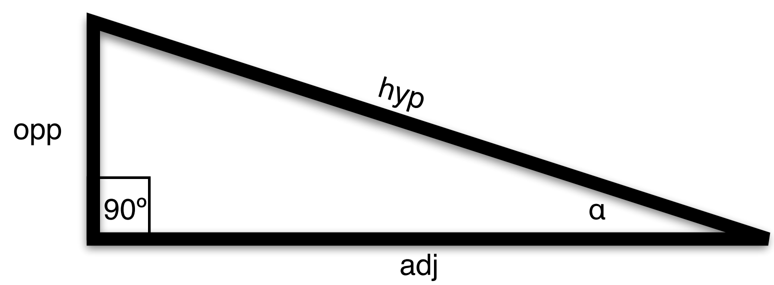

Figure 1: Right angle triangle.

$$\sin\alpha = \frac{\text{opp}}{\text{hyp}} \qquad \cos\alpha = \frac{\text{adj}}{\text{hyp}} \qquad \tan\alpha = \frac{\text{opp}}{\text{adj}}$$

From Pythagoras, we know that:

$$\textbf{hyp}^2 = \textbf{opp}^2 + \textbf{adj}^2$$

Small Angle Approximation:

In Astronomy, most of the angles we measure are extremely small. E.g. try to draw a right angle triangle with an angle α=1'. It should be very difficult indeed. When you try to do this, could you tell the difference in length between the adjacent length and the hypothenuse? In fact, the the previous problem, the difference between the hypothenuse and the adjacent is less than 0.02%. What this means is that whenever we work with small angles we can use a simplified trigonometric formula:

(size or height of object) ≈ (distance to object) × (angular size of object in radians)

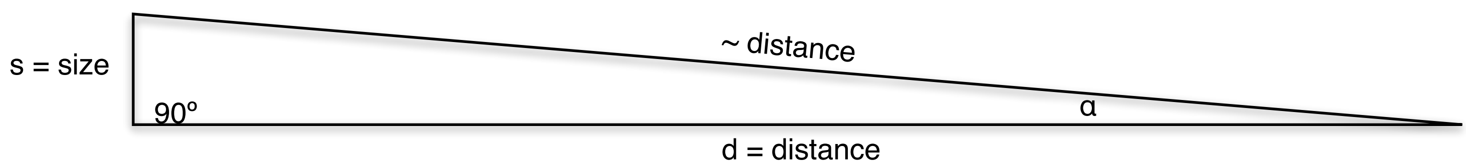

Figure 2: Right angle triangle for small angle α.

$$s \approx d \times \alpha \text{ (radians)}$$

Beware that the angle has to be in radians and the height or size of the object and the distance to the object will be in the same physical units (both km, both miles, both parsec, etc.).

Problem:

12) Imagine you face STB. If you stand with your nose up against the building, it will occupy your entire view. As you back away, the building appears to get smaller, it covers a smaller and smaller angle or angular size even though the size of the building has not changed at all. You can see that your distance to the STB, the actual size of the building and the angular size all work together so that you only need to know two of them to determine the third. Imagine that the STB subtends an angle of 1 degree. You know its height is 20 m. How far are you from the building? (Draw a triangle and make the correspondence between the generic angles in Figure 1 with those relevant to this problem).

13) Imagine you see a building far away with an angular size of 1 degree. If you are 10km from the building, how tall is the building? Calculate the values of the hypothenuse, the adjacent and opposite lengths.