Probability

$P(A)$ = "probability of event A"

$P(A|B)$ = "probability of event A given event B"

"given"

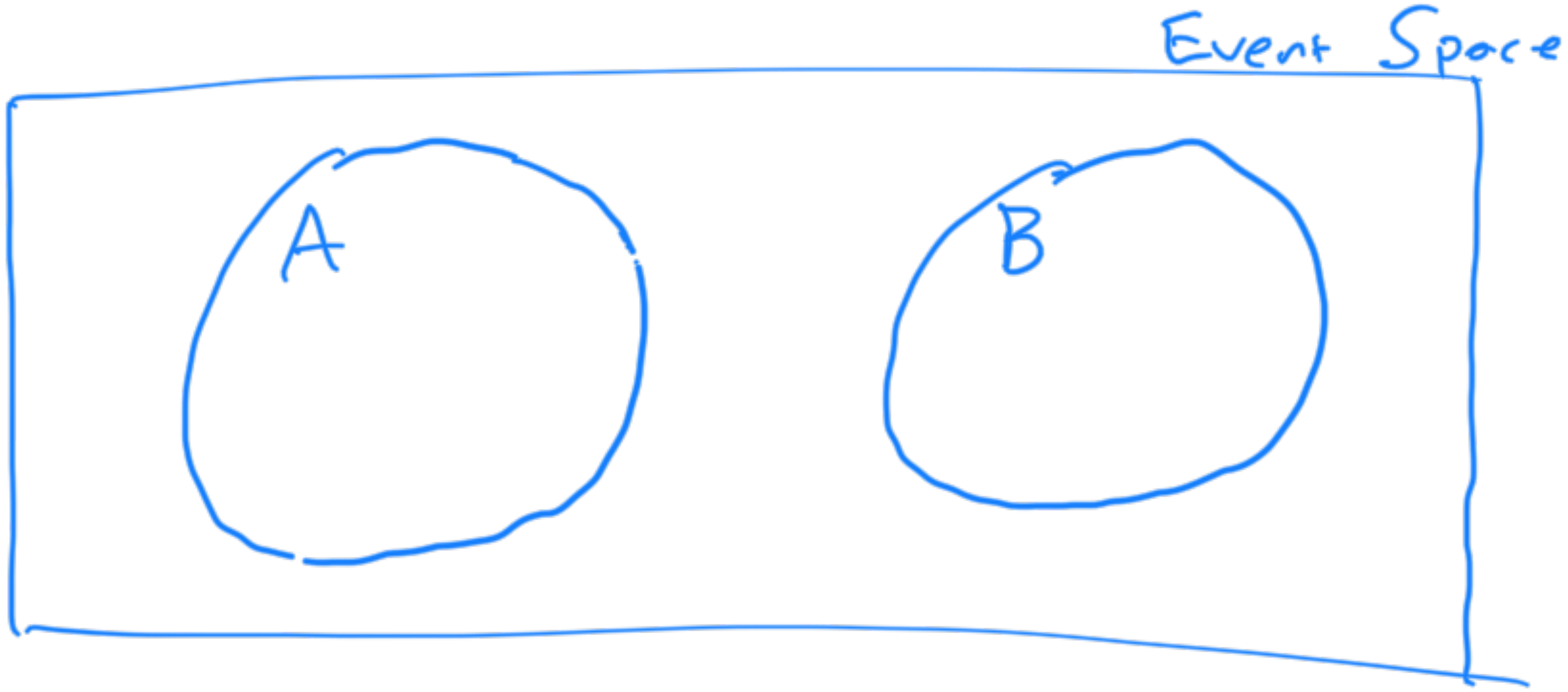
$P(\bar{A})$ = "probability of event A not occurring"

Basic Rules:

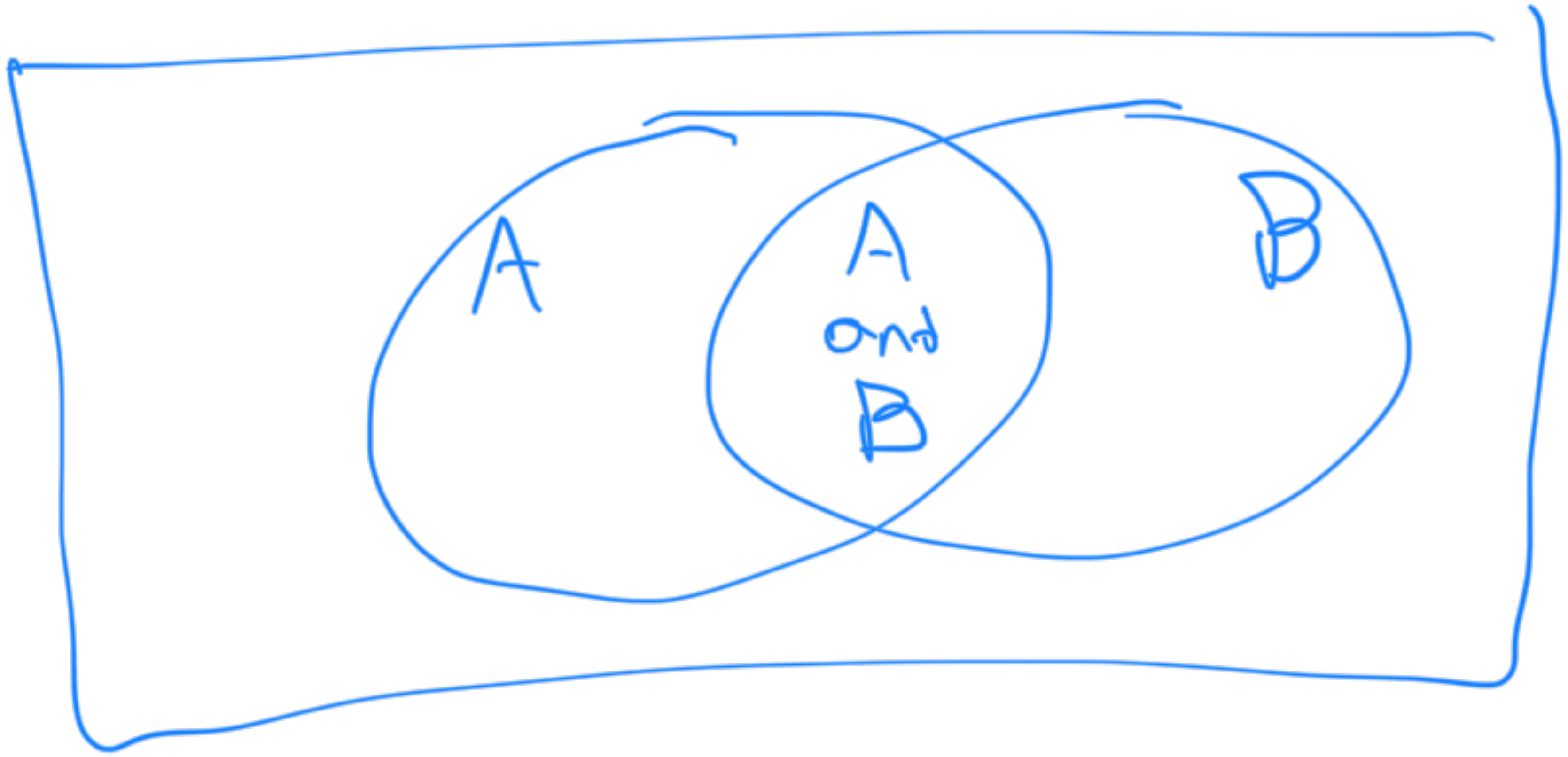
* sum of all probabilities in an event space $= 1$
* $0 \le P(x) \le 1$
* $P(\bar{A}) = 1 - P(A)$

  $P(A) = 1 - P(\bar{A})$
* $P(A \text{ or } B) = P(A) + P(B)$

  iff A and B are mutually exclusive



* $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$



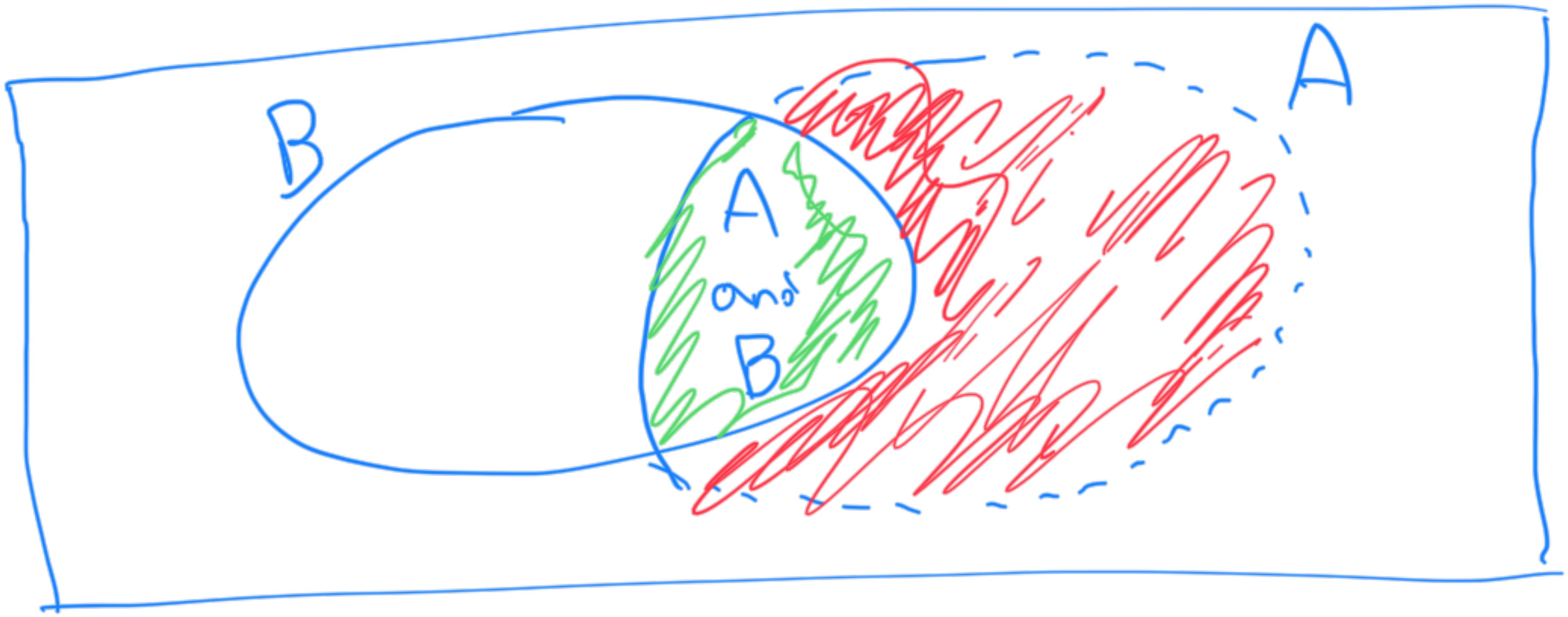
* $P(A \text{ and } B) = \underline{P(A)P(B|A) =} P(B)P(A|B)$

* $P(A \text{ and } B) = P(A)P(B)$

iff A and B are independent

* Bayes' Rule:

$$P(A|B) = \frac{P(A \text{ and } B)}{P(B)}$$



🟩 : event A that is relevant given B

🟥 : event A that is irrelevant given B

Email Spam Categorization Using Probability

Step 1
In Excel:

Text	Spam?
Buy this pill to ...	1
Free lottery Tickets! Just ...	1
Can I send you $1 million? ...	1
...	1
...	1
...	1
Dear Peter, Can you help me with my HW?	0
Check out this concert on ...	0
...	0
...	0

(1 = spam,
0 = not spam)

Step 2

Convert text into numerical representation:

1 is word appears in text
0 is word doesn't appear in text

(this table doesn't correspond to step 1's table)

hello	vicodin	. . . .	Spam?
0	1		1
1	1		1
0	0		1
1	1		1
1	1	. . .	1
0	0		1

1	0		1
1	0		0
1	1		0
1	1		0
			0

Step 3

Calculate the relevant probabilities for Bayes' Rule

$$P(A|B) = \frac{P(A \text{ and } B)}{P(B)} = \frac{\overbrace{P(B|A)}\,\overbrace{P(A)}}{\underbrace{P(B)}}$$

$$P(\text{Spam}|\text{each word}) = \frac{\overbrace{P(\text{word 1}|\text{spam})} \cdot \overbrace{P(\text{word 2}|\text{spam})} \cdot \ldots \cdot \overbrace{P(\text{Spam})}}{P(\text{seeing all words})}$$

$$P(\text{not Spam}|\text{each word}) = \frac{\overbrace{P(\text{word 1}|\text{not spam})} \cdot \overbrace{P(\text{word 2}|\text{not spam})} \cdot \ldots \cdot \overbrace{P(\text{not Spam})}}{P(\text{seeing all words})}$$

$P(\text{seeing all words})$

To categorize as spam, $P(\text{spam} \mid \text{each word}) > P(\text{not spam} \mid \text{each word})$

To categorize as not spam, $P(\text{spam} \mid \text{each word}) \leq P(\text{not spam} \mid \text{each word})$

$$P(\text{"hello"} \mid \text{spam}) = \frac{\text{number of spam emails with "hello"}}{\text{number of spam emails}} = 50\%$$

$$P(\text{"Vicodin"} \mid \text{spam}) = 4/6 = 66.7\%$$

$$P(\text{spam}) = 6/10 = 60\%$$

$$P(\text{"hello"} \mid \text{not spam}) = 4/4 = 100\%$$

$$P(\text{"Vicodin"} \mid \text{not spam}) = 2/4 = 50\%$$

$$P(\text{not spam}) = 40\%$$

Step 4

Use these probabilities to categorize new, unseen emails.

Example new email: "Hello! Buy my vicodin."

$$P(\text{spam} \mid \text{words}) \propto P(\text{"hello"} \mid \text{spam})\, P(\text{"vicodin"} \mid \text{spam})\, P(\text{spam}) = 10\%$$

$$P(\text{not spam} \mid \text{words}) \propto P(\text{"hello"} \mid \text{not spam})\, P(\text{"vicodin"} \mid \text{not spam})\, P(\text{not spam})$$

$$= 100\% \cdot 50\% \cdot 40\% = 20\%$$

Since $P(\text{spam} \mid \text{words}) > P(\text{not spam} \mid \text{words})$, put email in spam folder.

Example new email: "Hello! Let's go to lunch."

$$P(\text{spam} \mid \text{words}) \propto P(\text{"hello"} \mid \text{spam})\, P(\text{not "vicodin"} \mid \text{spam})\, P(\text{spam})$$

$$= 50\% \cdot 33.3\% \cdot 60\%$$

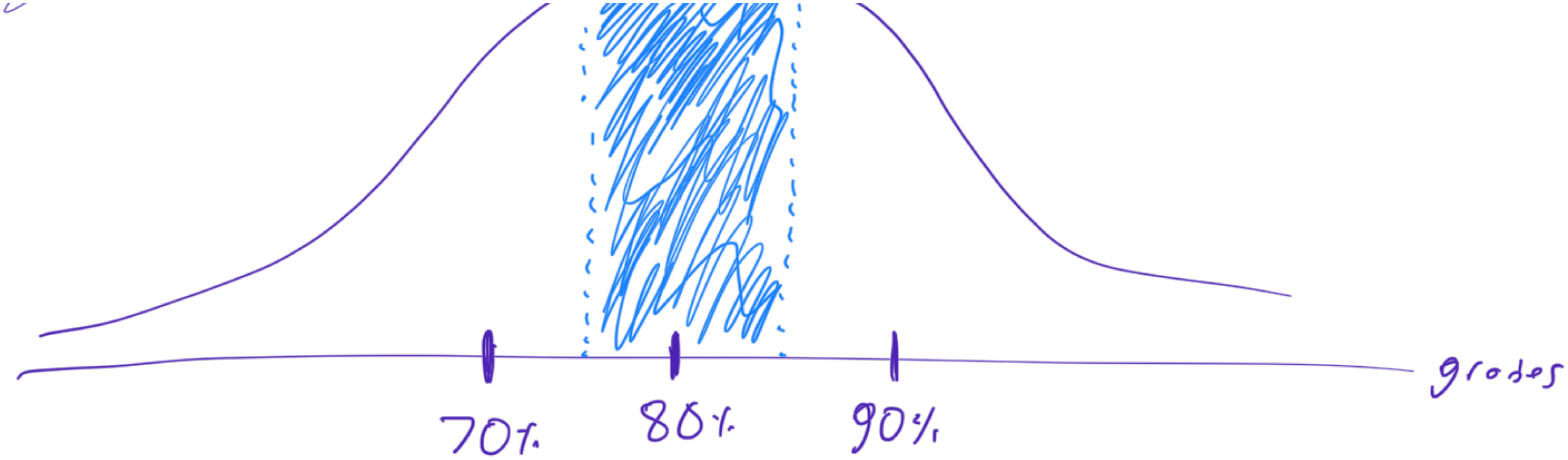
$$= 10\%$$

$$P(\text{not spam} \mid \text{words}) \propto P(\text{"hello"} \mid \text{not spam})\, P(\text{not "vicodin"} \mid \text{not spam}) \cdot P(\text{not spam})$$

$$= 100\% \cdot 50\% \cdot 40\%$$

$$= 20\%$$

Since $P(\text{not spam} \mid \text{words}) > P(\text{spam} \mid \text{words})$, don't put email in spam folder



$$P(75\% < \text{grade} < 85\%)$$

$$= \int_{75}^{85} \text{Bell Curve } dx$$