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Lecture 10: Introduction to Probability

Introduction

In statistics we never "prove" anything. All statistics allow us to do is make a decision with a certain amount of confidence. We are NEVER 100% confident we are right! YES! It's true. Much of our society is built upon the notion of 'scientific proof" and that is based upon probability! Guess what? There is a 1 in 20 chance that much of it is wrong! If you want to understand that STAY TUNED!

Types of Probability

Intuitive

Intuitive probability is a subjective guess based upon a hunch. It is not scientifically based. When ever you make an "educated guess" based upon your gut feeling to make a probability statement you are using this sort of probability. Example: *I think there is about a 20% chance that the Raiders will win the Superbowl this year.* So it's probability statement that is based upon a guess or a hunch.

A Priori

That term is Latin for something like "before hand." Basically in certain situations we know the probability of something happening in advance. Most games in casinos are *a priori* probability. For example, In craps I think the casinos pay 15:1 odds if your roll a "hard eight" [two fours]. The probability of that a happening is known in advance [by the way it's 36:1, that's why they can afford to build roller coasters inside of hotels in Las Vegas]. But any game of chance where the odds are known in advance is *a priori* probability: roulette wheels, keno [or bingo], slot machines are all this kind of probability.

So with *a priori* probability you know the probability of something happening before hand: coin toss, picking an ace of spades from a standard deck of cards, rolling snake eyes with dice, rolling a certain number with a single die, etc.

Empirical [or *a postori*]

This type of probability is sometimes called *A POSTORI*, which is also Latin, that means something like "after the fact" To make a probability statement using this we have to do a study to determine the actual probability. So after the study has been completed, then you know what the probability is.

So pretend the American Heart Association says something like "a typical American has a 50% chance of being overweight" or "an overweight male has a 40% greater chance of having heart disease." Those types of probability statements were possible because someone did a study of obesity and heart disease in the US population. (By the way those are not real probabilities, I made them up.)

So, with this type of probability, we determine the probability of something occurring after the study has been completed. In other words, a study is done that figures out the probability.

Simple Probability = one event and one event only

- It is based upon 100% or continuum from 0 to 1. 1 it will always happen 0 never happen. 0.5 means it will happen 50% of the time.

- It is expressed as fraction or decimal -- # of successes/ total # of possible outcomes: **coin: ½** or .5 or 50% die**:** 1/6 or .16 or 16%

Concepts and Terms for probability concepts

P (E) = "probability of Event"

sample space = set of all possible outcomes. (The number of possible outcomes in the sample space goes on "the bottom" of the fraction.)

For a fair six sided die the sample space ={1, 2, 3, 4, 5, 6}. These are all the possible outcomes if you throw a fair six sided die. (By the way a die is "one dice.")

event = the event is the thing you "want" to happen and it is always a subset (or portion) of sample space. (The number of possible outcomes in the event goes on "the top" of the fraction.)

If the event was "throwing an odd number from a fair six sided die," the event= {1, 3, 5}

So let's use the terminology: what is the probability of throwing an odd from a fair six-sided die?

P(odd number)=?

Sample space = {1, 2, 3, 4, 5, 6}. There are six possible outcomes so 6 goes on the bottom of the fraction.

event = what we want to happen = odd number = {1, 3, 5} There are 3 possible outcomes in the event so 3 goes on top of the fraction

P(odd number)= event/sample space = 3/6 3/6 is fraction that can be reduced to ½. So

P(odd number)=3/6 or ½ or 0.5 or 50%

simple event = this is the type of probability event that can't be broken down further. A simple event from a throw of a six sided fair die would be "What's the probability of rolling a 1 from a fair six sided die?" The event = {1}. Consider a coin toss where you have heads on one side of coin and tails on the other. A simple event from a coin toss would be one of the two possibilities: heads or tails BUT NOT BOTH!!!! So THERE IS NOT A "SIMPLE EVENT" from the example of "an odd number from a single six sided die" above!

Expected value and games of chance in Las Vegas

Explained in plain English, the expected value is a theoretical average if there were an infinite number of trials. Many times the expected value = the population mean.

So for example, you can bet on 36 numbers in roulette right? Half are red (18) and half (18)are black But there are two greens you can't bet on 0 and 00. The total number of spaces is 38 on a roulette wheel. So let's figure out the probability of winning if you bet on red. (The probability of winning on black are the same as there are an equal number black and red slots to bet on.)

P(red) =18/38 or 0.47 or 47%. As such if you keep playing and betting on a single color, over time, you will win less than half of the time. That means over time you lose more than you win. If you bet single numbers the odds are even worse over time.

What this means in practice, is the longer a person plays a game of chance the more likely they are to "revert to the mean." In essence, the longer you play a game based upon the laws of probability, the greater your chances are their results will revert to the mean. So for example you might imagine that if we took a coin and said 1=heads and 2=tails. If you flipped this coin ten times you might get more heads than tails. In fact the other day I had a student flip ten heads in a row in class – a very unlikely event. But the longer you flip the coin, the closer and closer you will get to having and equal number of heads and tails.

What does this mean in Las Vegas style gambling? Well you might beat the odds of a game of chance for a little while, but over time, Vegas will win. That's why they design casinos to keep you in them as long as possible: no windows, hard to find your way out, feed you free drinks, etc. The longer you play slots, roulette, or keno, [or even video card games for that matter] the greater the chances the "house wins." That's why Vegas can afford to build a roller coaster, or Paris, or Italy inside of a building in the middle of a desert.

The expected value is a theoretical average over time. It does not mean every single trial of a game of chance will meet the theoretical average, but it means over time, the outcomes of games of chance tend towards the theoretical average. Over time, you lose and the "house" wins in Vegas baby. That's why your secrets may stay in Vegas, but so does your money.

Complex Probability (prob. of compound event in book)= 2 or more events

both events A and B happen = P(A and B) = P(A) X P(B) *(REQUIRES INDEPENDENCE)*

Independence means

Two events are independent when occurrence of one event doesn't influence occurrence of the other. ex: have two die and throw at same time or two coins and toss at same time

ex 1: prob. of rolling "snake eyes" with a pair of six sided fair dice: P(1 and 1)= P(1) X P(1) = 1/6 x 1/6 = 1/36

ex 2: prob. of getting two heads when you toss two fair coins at one time: P(HEADS and HEADS) = P(HEADS) x P(HEADS) = ½ x ½=1/4

ex 3: Use a standard 52 deck of cards and pick 1 card return to deck, shuffle, and pick again. What is the probability of getting the king of hearts both times? P (king of hearts and king of hearts)= P(king of hearts) x P (king of hearts)= 1/52x1/52 = 1/2704

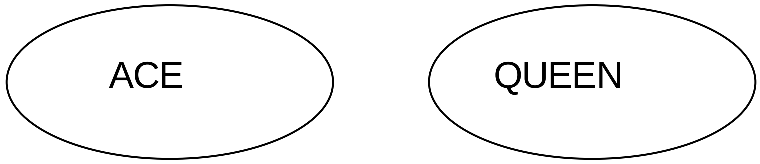
either event A or B happens = P(A or B) =P(A) + P(B) ***REQUIRES EVENTS TO BE MUTUALLY EXCLUSIVE***

Mutually Exclusive means

Two events are mutually exclusive of each other when the do not share any common outcomes. *(If the two events are mutually exclusive they are also independent.)* Below is an example of two events being mutually exclusive:

regular 52 card deck. P(Ace or Queen)

MUTUALLY EXCLUSIVE



example 1: picking an Ace or Queen from a regular 52 card deck. P(Ace or Queen)=

P(Ace)+P(Queen)= 4/52 + 4/52 = 8/52

example 2: probability of getting a 2 or a 3 from one roll of a fair six sided die =

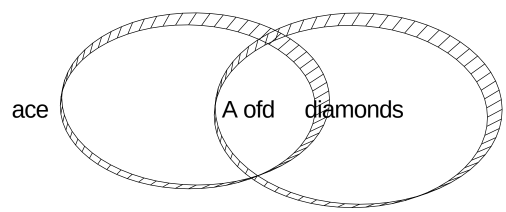
p(2) + p(3) = 1/6 + 1/6 = 2/6 or 1/3

- **either event A or B happens & *NOT MUTUALLY EXCLUSIVE* = P(A or B) =P(A) + P(B) - P(A&B)**

ex. example: regular 52 card deck. P(Ace or Diamond)

well there is an Ace of Diamonds so there is some overlap and these are NOT mutually exclusive.

NOT MUTUALLY EXCLUSIVE



P(Ace or Diamond) =P(Ace) + P(Diamond) - P(Ace&Diamond)
= 4/52 + 13/52 - (4/52 x 13/52)
= 17/52 - 52/2704

to get common denominator multiply both top and bottom of first fraction by 52

=884/2704 - 52/2704
=832/2704

reduce fraction and divide both top and bottom of fraction by 16

=16/52

- Won't do conditional probabilities in this class…