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| Statistics<br>16b_practice.pdf | Michael Hallstone, Ph.D.<br>hallston@hawaii.edu |

# Lecture 16: Practice Problems for Estimating Parameters (Confidence Interval Estimates of the Mean)

## Practice

Everything that appears in these lecture notes are fair game for the test. They are the best "study guide" I can provide. It is impossible to provide a "list" that is more comprehensive than the lecture notes above. However, here are a few additional practice exercises or practice concepts.

### *1. What is the mean number of days needed to complete "successful" drug treatment program for released prisoners?*

Pretend a public administrator is trying to determine a budget for a state program that is going to offer drug treatment to people released from prison. In order to figure out how much a program might cost, she needs to know about how long people it takes people to "graduate" successfully from drug treatment programs. So pretend she collects some data from a random sample of successful graduates and finds:

$\bar{x}$ =90 days s =10 days n=100. The population standard deviation is unknown.

**Construct a 95% confidence interval estimate of the population mean days required for successful completion of drug treatment. Then say what the numbers mean in plain English**

**Answer:**

$\bar{x} - z\,\hat{\sigma}\bar{x} < \mu < \bar{x} + z\,\hat{\sigma}\bar{x}$ where $\hat{\sigma}\bar{x} = \dfrac{s}{\sqrt{n}}$

$$\hat{\sigma}\bar{x} = \frac{s}{\sqrt{n}} = \frac{10}{\sqrt{100}} = \frac{10}{10} = 1$$

$$\bar{x} - z\,\hat{\sigma}\bar{x} < \mu < \bar{x} + z\,\hat{\sigma}\bar{x}$$

$90 - (1.96)\,1 < \mu < 90 + (1.96)$

$90 - 1.96 < \mu < 90 + 1.96$

$88.04 < \mu < 91.96$

The administrator can be 95% confident that the mean time it takes the population of released prisoners to successfully complete drug treatment is between 88.04 and 91.96 days.

## *2. What is the mean pharmacy co-payment of clients served by a public health clinic?*

A health care administrator receives a new federal grant to help pay for pharmacy co-payments at a rural health clinic. She figures she will base the new co-payment on the mean co-payment paid by the current clients at the health clinic. She takes a random sample of 49 clients and finds

$\bar{x}$ =\$5 s =\$10 n=49. The population standard deviation is unknown.

**Construct a 99% confidence interval estimate of the mean co-payment of clients at the rural health clinic . Then say what the numbers mean in plain English**

**Answer:**

$\bar{x} - z\,\hat{\sigma}\bar{x} < \mu < \bar{x} + z\,\hat{\sigma}\bar{x}$ where $\hat{\sigma}\bar{x} = \frac{s}{\sqrt{n}}$

$$\hat{\sigma}\bar{x} = \frac{s}{\sqrt{n}} = \frac{10}{\sqrt{49}} = \frac{10}{7} = 1.43$$

$\bar{x} - z\,\hat{\sigma}\bar{x} < \mu < \bar{x} + z\,\hat{\sigma}\bar{x}$

5- (2.575) 1.43 < μ< 5 + (2.575) 1.43

5−3.68< μ< 5+3.68

1.32< μ< 8.68

The administrator can be 99% confident that the mean copayment of the population at the pharmacy is between \$1.32 and \$8.68.

## *3. What is the mean time (in minutes) spent in line at customer service line at a C&C City Hall office?*

Pretend a public administrator who oversees all the satellite city halls on Oahu receives a directive from the mayor that customers should not be made to wait more than 5 minutes in line while waiting for service. The administrator wants to figure out if his current staffing levels are meeting this new requirement he takes a random sample of the wait time of 144 customers at all of the satellite city halls and finds:

$\bar{x}$ = 5 min s =10 min n=144. The population standard deviation is unknown.

Construct a 90% confidence interval of the mean wait time of customers in the customer service line. Can the administrator be 90% confident that the mean wait time is under 5 minutes?

**Answer:**

$\bar{x} - z\,\hat{\sigma}\bar{x} < \mu < \bar{x} + z\,\hat{\sigma}\bar{x}$ where $\hat{\sigma}\bar{x} = \frac{s}{\sqrt{n}}$

$$\hat{\sigma}\bar{x} = \frac{s}{\sqrt{n}} = \frac{10}{\sqrt{144}} = \frac{10}{12} = .83$$

$$\bar{x} - z\,\hat{\sigma}\bar{x} < \mu < \bar{x} + z\,\hat{\sigma}\bar{x}$$

5- (1.645) .83 < μ< 5 + (1.645) .83
5−1.37< μ< 5+1.37
3.63< μ< 6.37

The administrator cannot say with 90% confidence that his mean wait time is less than 5 minutes as his upper interval is above 5 seconds. However, the administrator could say , with 90% confidence, that the mean wait time is between 3.63 minutes and 6.37 minutes.

### *4. What is the mean age of clients served at one branch of a YMCA agency?*

Pretend the administrator at the non-profit YMCA wants to decide which sort of recreational equipment to buy for the Waianae Branch YMCA. By observation it appears that different YMCA branches have clients of different mean age: the branch across the street from Ala Moana shopping center seems to serve mostly adults while the Waianae branch seems to serve mostly kids who stop by after school. The administrator takes a random sample of 25 clients at the Waianae branch and finds:

$\bar{x}$ = 14 years s=8 years n=25. The population is standard deviation is unknown but the size of the population is 122.

Construct a 95% confidence interval of the population mean age of the clients served at the Waianae Branch of the YMCA. Explain what the answer means in plain English.

Answer:

$$\hat{\sigma}\bar{x} = \frac{s}{\sqrt{n}}\sqrt{\frac{N-n}{N-1}} = \frac{8}{\sqrt{25}}\sqrt{\frac{122-25}{122-1}} = \frac{8}{5}\sqrt{\frac{97}{121}} = 1.6\sqrt{.80} = (1.6)(.895) = 1.43$$

$$\bar{x} - t_{\alpha/2}\hat{\sigma}\bar{x} < \mu < \bar{x} + t_{\alpha/2}\hat{\sigma}\bar{x}$$

14- (2.064)(1.43) < μ< 14+ (2.064)(1.43)

14-2.95152< μ< 14+2.95152

11.05< μ< 16.95

The YMCA administrator can be 95% confident that the mean age of the population of Waianae YMCA clients is between 11.05 and 16.95 years of age.

### *Even More Practice Problems*

For the following problems assume population standard deviation is unknown the population is infinite and construct a 95% confidence interval. Use the following formula for all

$$\bar{x} - z\,\hat{\sigma}\bar{x} < \mu < \bar{x} + z\,\hat{\sigma}\bar{x} \quad \text{where } \hat{\sigma}\bar{x} = \frac{s}{\sqrt{n}}$$

| x-bar | n | s |
|---|---|---|
| 10 | 49 | 7 |

| x-bar | n | s |
|---|---|---|
| 20 | 64 | 8 |
| | | |
| **x-bar** | **n** | **s** |
| 30 | 64 | 8 |

| x-bar | n | s |
|---|---|---|
| 40 | 81 | 9 |

| x-bar | n | s |
|---|---|---|
| 50 | 100 | 10 |

| x-bar | n | s |
|---|---|---|
| 60 | 121 | 11 |

| x-bar | n | s |
|---|---|---|
| 70 | 144 | 12 |
| | | |

***Answers are below…***

## *Answers to above*

| x-bar | n | s | sq. root of n | s/sqroot n | z score |
|---|---|---|---|---|---|
| 10 | 49 | 7 | 7 | 1 | 1.96 |
| | | | | | |
| **x-bar -** | **z*SE** | **<pop mean<** | **xbar +** | **z*SE** | |
| 10 | 1.96 | <pop mean< | 10 | 1.96 | |
| | 8.04 | <pop mean< | 11.96 | | |

| x-bar | n | s | sq. root of n | s/sqroot n | z score |
|---|---|---|---|---|---|
| 20 | 64 | 8 | 8 | 1 | 1.96 |
| | | | | | |
| **x-bar -** | **z*SE** | **<pop mean<** | **xbar +** | **z*SE** | |
| 20 | 1.96 | <pop mean< | 20 | 1.96 | |
| | 18.04 | <pop mean< | 21.96 | | |

| x-bar | n | s | sq. root of n | s/sqroot n | z score |
|---|---|---|---|---|---|
| 30 | 64 | 8 | 8 | 1 | 1.96 |
| | | | | | |
| **x-bar -** | **z*SE** | **<pop mean<** | **xbar +** | **z*SE** | |
| 30 | 1.96 | <pop mean< | 30 | 1.96 | |
| | 28.04 | <pop mean< | 31.96 | | |

| x-bar | n | s | sq. root of n | s/sqroot n | z score |
|---|---|---|---|---|---|
| 40 | 81 | 9 | 9 | 1 | 1.96 |
| | | | | | |
| **x-bar -** | **z*SE** | **<pop mean<** | **xbar +** | **z*SE** | |
| 40 | 1.96 | <pop mean< | 40 | 1.96 | |
| | 38.04 | <pop mean< | 41.96 | | |

| x-bar | n | s | sq. root of n | s/sqroot n | z score |
|---|---|---|---|---|---|
| 50 | 100 | 10 | 10 | 1 | 1.96 |
| | | | | | |

| **x-bar -** | **z*SE** | **<pop mean<** | **xbar +** | **z*SE** | |
|---|---|---|---|---|---|
| 50 | 1.96 | <pop mean< | 50 | 1.96 | |
| | 48.04 | <pop mean< | 51.96 | | |

| **x-bar** | **n** | **s** | **sq. root of n** | **s/sqroot n** | **z score** |
|---|---|---|---|---|---|
| 60 | 121 | 11 | 11 | 1 | 1.96 |
| | | | | | |
| **x-bar -** | **z*SE** | **<pop mean<** | **xbar +** | **z*SE** | |
| 60 | 1.96 | <pop mean< | 60 | 1.96 | |
| | 58.04 | <pop mean< | 61.96 | | |

| **x-bar** | **n** | **s** | **sq. root of n** | **s/sqroot n** | **z score** |
|---|---|---|---|---|---|
| 70 | 144 | 12 | 12 | 1 | 1.96 |
| | | | | | |
| **x-bar -** | **z*SE** | **<pop mean<** | **xbar +** | **z*SE** | |
| 70 | 1.96 | <pop mean< | 70 | 1.96 | |
| | 68.04 | <pop mean< | 71.96 | | |