Lecture 17a: P-values

Introduction

The purpose of this lecture is to introduce you to the concept of p-values. We will learn how to compute p-values by hand. It is necessary to do these problems to understand the p value that is automatically spit out by SPSS. It is easy to make SPSS spit out a p-value. It is much harder to know what a p-value actually means in plain English.

See lecture 17c: SPSS output (17c_SPSS.pdf) to learn how to use SPSS to compute a p value.

What is a p-value

How to use a p-value to make the statistical decision in step 6 of whether to reject or fail to reject the null hypothesis

How to compute a p-value by hand

How to use SPSS statistics software to recognize p-values for hypothesis tests of means or t-tests

What is p-value?

The p-value is the probability of Type I error. Type I error is the probability of rejecting a correct null hypothesis. However I prefer plain English. The p-value is the probability of incorrectly rejecting the null hypothesis. Or the p-value is the probability of rejecting a null hypothesis when in fact it is ‘true.’ Or the p-value is the chance of error you will have to accept if you want to reject the null hypothesis. All of these are different ways of explaining p-value in plain English.

Examples of p-values

Examples:
• a p-value of .01 means there is a 1% chance that we will incorrectly reject the null hypothesis. Or that we could reject the null hypothesis with a 1% chance of error.
• a p-value of .04 means there is a 4% chance that we are incorrectly rejecting the null hypothesis. Or that we could reject the null hypothesis with a 4% chance of error.
• a p-value of .10 means there is a 10% chance that our decision to reject the null hypothesis was in error. Or that we could reject the null hypothesis with a 10% chance of error.

Using p-value to make a decision in (place of) step 7

In step 7 you make the decision of whether or not to reject the null hypothesis. Recall in step 2 of the 7 steps you set alpha, or the amount of error you are willing to accept if you reject the null hypothesis.

Using a p-value, one can make the decision to reject or fail to reject the null hypothesis.

If \( p > \alpha \) then FAIL TO REJECT the null hypothesis.

If \( p < \alpha \) then REJECT the null hypothesis.

Computing p-value by hand

**NOTE!** We will not compute p value by hand when \( n < 30 \) (and we use t table) in this class. This is because of the way the t-table in the book is structured. A better t table would allow for hand computations. But in this class, when we use the t table we will rely on SPSS to compute the p-value.

**p-value for a two tailed test**

To compute a p-value by hand all you do is find the area “outside” of the test ratio value from step 6 in ‘normal curve’ – that is your p-value. There are two areas “outside” of your test ratio from step 6 – one on each side of the normal curve.

In the first example of lecture 17: one sample hypothesis testing (17_one_sample_t_test.pdf) we examined the mean LSAT scores with the following

Step 1: \( H_0: \mu = 1200 \)

\( H_1: \mu \neq 1200 \)

step 4: reject null if \( TR > 1.96 \) or \( TR < -1.96 \)

Step 6: \( TR = -2.0 \)

The p-value is the area to the “outside” of the z-scores of -2.0 and 2.0.
We find that the area for $Z(2.0) = .4772$. By using the properties of the $z$-distribution we know that $0.50 - .4772 = .0228$ (or 2.28%). So there is 2.28% in BOTH tails of the curve so $0.0228 + 0.0228 = 0.046$ or 2.28% + 2.28% = 4.56%. The p-value for a TR $= 2.0 = 0.0456$. With this TR score there is a 4.6% chance of incorrectly rejecting the null hypothesis. Or there is a 4.56% chance that the null hypothesis is correct. So in this case $p = 0.0456$ or 4.56% $p < \alpha$ so you can reject the null hypothesis.

**Explaining what a p-value means in plain English regarding the statistical decision made in Step 7**

So in this case $p = 0.0456$ or 4.56%. In plain English, this means you can reject the null hypothesis and conclude that the mean LSAT score of prep course graduates is not equal to 1200 with a 4.56% chance of being wrong. Or conversely, you can reject the theory that the mean LSAT score of prep course graduates is equal to 1200 with a 4.56% chance of being wrong.
**p-value for a one tailed test**

Here you need to pay attention to whether or not your test ratio from step 6 (TR) falls in the tail that has the rejection region. Recall that in a one-tailed test the “arrow” in the bottom or alternative hypothesis “points” to the rejection region.

**p-value when rejection region is to the left or negative side of the curve**

Recall In the second example of *lecture 17: one sample hypothesis testing (17_one_sample_t_test.pdf)* we examined the mean LSAT scores with the following

Step 1:  \( H_0: \mu \geq 1200 \)
\[ \quad H_1: \mu < 1200 \]

Step 4: Reject null if TR < -1.645 otherwise fail to reject null.

Step 6:  \( TR = -2.0 \)

In this case the p-value is the total area under the curve from the TR all the way out through the left or negative side of the curve *regardless of whether or not the TR is negative or positive!*

\[ H_0: \mu \geq 1200 \]
\[ H_1: \mu < 1200 \]

The p-value is the area to the left of the z-score -2.0.
In step 6, TR= -2.0  z(2.0)=.4772  .50 - .4772 = .0228 (or 2.28%). Since we are only concerned with the negative side of the curve p=.0228 or p=2.28%. There is a 2.28% chance of incorrectly rejecting null. Or there is a 2.25% chance that the null hypothesis is correct.

Explaining what a p-value means in plain English regarding the statistical decision made in Step 7

The SPSS p value for this TR value would be p=.0228 or 2.28%. In plain English this means you can reject the null can conclude that mean LSAT score of prep course graduates is less than 1200 with a 2.28% chance of being wrong.

p-value when rejection region is to the right or positive side of the curve

In this case the p-value is the total area under the curve from the TR all the way out through the right or positive side of the curve regardless of whether or not the TR is negative or positive!

H₀: μ ≤ 1200
H₁: μ > 1200
Computing p when TR is in the CORRECT TAIL

p = .5 - (area of z table)

H₀: µ ≥ ?   H₁: µ < ? (Step 4: reject null if TR < -1.645 and Step 6: TR = -2.0)
Computing \( p \) when TR is in the INcorrect TAIL

\[ p = 0.5 + (\text{area of } z \text{ table}) \]

\( H_0: \mu \leq ? \quad H_1: \mu > ? \) (Step 4: reject null if TR < -1.645 and Step 6: TR = +2.0)

\[ TR = +2 \quad \text{Step 4: reject null if } TR < -1.645 \]

\[ p = 0.5 + 0.4772 = 0.9772 \quad \text{or 97.72\%} \]

TR=+2. \( 0.5 + 0.4772 = 0.9772 \) or 97.72%

\( p \)-values on SPSS

see the lecture “17c_SPSS.pdf”