Michael Hallstone, Ph.D.
hallston@hawaii.edu

Lecture 17b: Practice Problems for Lecture 17 "One Sample Hypothesis Test of Means"

Since lecture 17 was so long, I decided to put the practice problems in a separate document.

Practice

Everything that appears in these lecture notes is fair game for the test. They are the best "study guide" I can provide. It is impossible to provide a "list" that is more comprehensive than the lecture notes above. However, here are a few additional practice exercises or practice concepts.

1: Line waiting = 5 minute WHEN α =.01

Pretend the mayor instituted a program designed to lower the amount of time people wait in line to receive services at City Hall. The mayor wants to know if the mean waiting time is equal or not equal to 5 minutes. Test the hypothesis that the mean waiting time is equal to five minutes in the population. Or conversely see if your t-test can prove that the mean waiting time in the population is NOT EQUAL to five minutes. Pretend a study was done and found:

s=10 n=100 $\bar{x}$= 3 μ_{Ho}=5 minutes α =.01

Do the seven steps for a hypothesis test of mean or t-test. Then after step 7, compute the p-value by hand if possible (recall we do not compute the p-value by hand if we use the t-distribution).

2 : Line waiting =less than 5 minutes WHEN α =.01?

Pretend the mayor instituted a program designed to lower the amount of time people wait in line to receive services at City Hall. The mayor wants to know if the mean waiting time is equal or not equal to 5 minutes. Test the hypothesis that the mean waiting time in the population is GREATER THAN OR EQUAL to five minutes. Or conversely see if your t-test can prove that the mean waiting time in the population is LESS THAN to five minutes. Pretend a study was done and found:

s=10 n=100 $\bar{x}$= 3 μ_{Ho}=5 minutes α =.01

Do the seven steps for a hypothesis test of mean or t-test. Then after step 7, compute the p-value by hand if possible (recall we do not compute the p-value by hand if we use the t-distribution).

3 : Line waiting =less than 5 minutes WHEN α =.05?

Pretend the mayor instituted a program designed to lower the amount of time people wait in line to receive services at City Hall. The mayor wants to know if the mean waiting time in the population is equal or not equal to 5 minutes. Test the hypothesis that the mean waiting time in the population is GREATER THAN OR EQUAL to five minutes. Or conversely see if your t-test can prove that the mean waiting time in the population is LESS THAN to five minutes. Pretend a study was done and found:

s=10 n=100 $\bar{x}$= 3 μ_{Ho}=5 minutes α =.05

Do the seven steps for a hypothesis test of mean or t-test. Then after step 7, compute the p-value by hand if possible (recall we do not compute the p-value by hand if we use the t-distribution).

4 : scores higher than 100? WHEN α =.05?

Pretend the Department of Education (DOE) claims that increasing teacher training has resulted in higher mean test scores for students. The mean test score before the increased teacher training [hypothetical population mean] is = 100. The sample mean test score of students after the teacher training =103. Test the hypothesis that the mean test score in the population is LESS THAN OR EQUAL to 100. Or conversely see if your t-test can prove that the mean test score in the population is GREATER THAN 100. Pretend a study was done and found:

s=7 n=49 $\bar{x}$= 103 μ_{Ho}=100 α =.05

Do the seven steps for a hypothesis test of mean or t-test. Then after step 7, compute the p-value by hand if possible (recall we do not compute the p-value by hand if we use the t-distribution).

5 : scores higher than 100? WHEN α =.01?

Pretend the Department of Education (DOE) claims that increasing teacher training has resulted in higher mean test scores for students. The mean test score before the increased teacher training [hypothetical population mean] is = 100. The sample mean test score of students after the teacher training =103. Test the hypothesis that the mean test score in the population is LESS THAN OR EQUAL to 100. Or conversely see if your t-test can prove that the mean test score in the population is GREATER THAN 100. Pretend a study was done and found:

s=7 n=49 $\bar{x}$= 103 μ_{Ho}=100 α =.01

Do the seven steps for a hypothesis test of mean or t-test. Then after step 7, compute the p-value by hand if possible (recall we do not compute the p-value by hand if we use the t-distribution).

6 # of prior DUIs >0 WHEN α =.05?

I collected data from a random sample of 28 people arrested for DUI who refused to take a Blood Alcohol Content (BAC) test. It makes sense that these people refuse to take the test because they have previously been convicted of DUI. Test the hypothesis that the mean number of prior DUI convictions in the population is less than or equal to zero. Conversely, to a hypothesis test to prove that the mean number of prior DUI convictions in the population is greater than zero. You cannot compute the p-value by hand for this problem.

s=3.564 n=28 $\bar{x}$= 1.96 μ_{Ho}=0 α =.05

7: BAC higher than .08 WHEN α =.01?

It is against the law to drive with a BAC that is greater than 0.08. Below is data from a random sample of 475 people arrested for DUI. Do a hypothesis test to prove that mean BAC of DUI arrestees in the population is greater than 0.08. Conversely, test the theory that the mean BAC of DUI arrestees in the population is less than or equal to 0.08. As you will see in the answer section, you cannot compute this p-value by hand.

s=0.063 n=475 $\bar{x}$=0.13502 μ_{Ho}=0.08 α =.01

8: BAC higher than .10 WHEN α =.01?

Clearly the hypothesis test in #7 made little sense. Obviously, we would expect the mean BAC of DUI arrestees to be greater than 0.08, as that is the minimum BAC to be a “drunk driver.”

Do a hypothesis test to prove that mean BAC of DUI arrestees in the population is greater than 0.10. Conversely, test the theory that the mean BAC of DUI arrestees in the population is less than or equal to 0.10. As you will see in the answer section, you cannot compute this p-value by hand.

s=0.063 n=475 $\bar{x}$=0.13502 μ_{Ho}=0.10 α =.01

Practice Problem Answers

1: Line waiting = 5 minute WHEN α =.01

s=10 n=100 $\bar{x}$= 3 μ_{Ho}=5 minutes α =.01

Step 1: State the null and alternative hypothesis (H_0 and H_1).
H_0: μ = 5
H_1: $\mu \neq$ 5
Step 2: State level of significance or α "alpha."
For this example we'll use α =.01

Step 3: Determine the test distribution to use – z or t.
For this example, although the population parameters are unknown, we have a sample size bigger than 30 so we use z
Step 4 and 5

In this case, we have two-tailed test so we split the 1% up – ½ in each tail. That translates to z(2.575)=.4950.
Step 5: State the decision rule.
Reject the null if the TR >2.575 or TR< -2.575, otherwise FTR.

Step 6: Perform necessary calculations on data and compute TR value.

$$\sigma_x = \frac{s}{\sqrt{n}} = \frac{10}{\sqrt{100}} = \frac{10}{10} = 1$$

$$TR = \frac{\bar{x} - \mu_{Ho}}{\sigma_x} = \frac{3-5}{1} = \frac{-2}{1} = -2$$

Step 7: Compare TR value with the decision rule and make a statistical decision. (Write out decision in English! -- my addition)
-2 falls in fail to reject region Therefore we FAIL TO REJECT NULL and there is insufficient evidence to reject the theory that the mean waiting time is equal to five minutes with 99% confidence.

P-value in plain English
p=100%-95.44%=4.56% or p=.0456 p>α therefore fail to reject the null hypothesis. If you were to reject the null and reject the theory that the mean waiting time in the population is NOT equal to five minutes you would have to accept a 4.56% chance of being wrong. [Note the error or alpha in step 2 is .01 or 1%, which is why we failed to reject the theory in the null hypotheisis)

2 : Line waiting =less than 5 minutes WHEN α =.01?

s=10 n=100 $\bar{x}$= 3 μ_{Ho}=5 minutes α =.01

Step 1: State the null and alternative hypothesis (H_0 and H_1).
H_0: $\mu \geq 5$
H_1: $\mu < 5$

Step 2: State level of significance or α "alpha."
For this example we'll use α =.01

Step 3: Determine the test distribution to use – z or t.
For this example, although the population parameters are unknown, we have a sample size bigger than 30 so we use z
Step 4 and 5

In this case, we have one tailed test so all 1% goes into the LEFT or NEGATIVE tail. That translates to z(-2.325)=.4900.
Step 5: State the decision rule.
Reject the null if the TR< -2.325, otherwise FTR.

Step 6: Perform necessary calculations on data and compute TR value.

$$\sigma_x = \frac{s}{\sqrt{n}} = \frac{10}{\sqrt{100}} = \frac{10}{10} = 1$$

$$\text{TR} = \frac{\bar{x} - \mu_{Ho}}{\sigma_x} = \frac{3-5}{1} = \frac{-2}{1} = -2$$

Step 7: Compare TR value with the decision rule and make a statistical decision. (Write out decision in English! -- my addition)
-2 falls in fail to reject region. Therefore we FAIL TO REJECT NULL and there is insufficient evidence to reject the theory that the mean waiting time in the population is greater than or equal to five minutes with 99% confidence. Or there is insufficient evidence to reject the theory that the mean waiting time is greater than or equal to 5 minutes.
P-value
p=100%-97.72%=2.28% .0228 p>α therefore fail to reject the null hypothesis

If you were to reject the theory that the mean waiting time is less than five minutes you would have to accept a 2.28% chance of error. (Note: In step 2 we decided we would only accept a less than 1% chance of error as α =.01 and this is why we fail to reject the theory in the null hypothesis.)

3 : Line waiting =less than 5 minutes WHEN α =.05?

s=10 n=100 $\bar{x}$= 3 μ_{Ho}=5 minutes α =.05

Step 1: State the null and alternative hypothesis (H_0 and H_1).
H_0: $\mu \geq 5$
H_1: $\mu < 5$

Step 2: State level of significance or α "alpha."
For this example we'll use α =.05

Step 3: Determine the test distribution to use – z or t.
For this example, although the population parameters are unknown, we have a sample size bigger than 30 so we use z

Step 4 and 5

In this case, we have one tailed test so all 5% goes into the LEFT or NEGATIVE tail. That translates to z(--1.645)=.4500.

Step 5: State the decision rule.

Reject the null if the TR< -1.6455, otherwise FTR.

Step 6: Perform necessary calculations on data and compute TR value.

$$\sigma_x = \frac{s}{\sqrt{n}} = \frac{10}{\sqrt{100}} = \frac{10}{10} = 1$$

$$TR = \frac{\bar{x} - \mu_{Ho}}{\sigma_x} = \frac{3-5}{1} = \frac{-2}{1} = -2$$

Step 7: Compare TR value with the decision rule and make a statistical decision. (Write out decision in English! -- my addition)

-2 falls in the rejection region Therefore we REJECT NULL and CAN conclude that the mean waiting time in the population is less than five minutes with 95% confidence

P-value

p=100%-97.72%=2.28% or .0228 $p<\alpha$ therefore reject the null hypothesis

In plain English, this p-value means you reject the null and conclude that the mean waiting time in the population is less than five minutes with only a 2.28% chance of error (or with 97.72% chance of being correct). So here our alpha or acceptable error in step 2 was 5%-- thus we reject the null hypothesis and conclude the alternative.

4 : scores higher than 100? WHEN α =.05?

s=7 n=49 $\bar{x}$= 103 μ_{Ho}=100 α =.05

Step 1: State the null and alternative hypothesis (H_0 and H_1).

H_0: $\mu \leq 100$

H_1: $\mu > 100$

Step 2: State level of significance or α "alpha."

For this example we'll use α =.05

Step 3: Determine the test distribution to use – z or t.

For this example, although the population parameters are unknown, we have a sample size bigger than 30 so we use z

Step 4 and 5

In this case, we have one tailed test so all 5% goes into the RIGHT or POSITIVE tail. That translates to z(1.645)=.4500.

Step 5: State the decision rule.

Reject the null if the TR>1.6455, otherwise FTR.

Step 6: Perform necessary calculations on data and compute TR value.

$$\sigma_x = \frac{s}{\sqrt{n}} = \frac{7}{\sqrt{49}} = \frac{7}{7} = 1$$

TR = $\dfrac{\bar{x} - \mu_{Ho}}{\sigma_x} = \dfrac{103 - 100}{1} = \dfrac{3}{1} = 3$

Step 7: Compare TR value with the decision rule and make a statistical decision. (Write out decision in English! -- my addition)

+3 falls in the rejection region. Therefore we REJECT NULL and CAN conclude that mean test scores in the population are higher than 100 with 95% confidence

P-value in plain English

p=100%-99.87%=0.13% or .0013 $p<\alpha$ therefore reject the null hypothesis

In plain English, you reject the null and conclude that mean test scores in the population are higher than 100 with only a 0.13% chance of error (or with 99.87% chance of being correct).

5 : scores higher than 100? WHEN α =.01?

s=7 n=49 $\bar{x}$= 103 μ_{Ho}=100 α =.01

Step 1: State the null and alternative hypothesis (H_0 and H_1).

H_0: $\mu \leq 100$

H_1: $\mu > 100$

Step 2: State level of significance or α "alpha."

For this example we'll use α =.01

Step 3: Determine the test distribution to use – z or t.

For this example, although the population parameters are unknown, we have a sample size bigger than 30 so we use z

Step 4 and 5

In this case, we have one tailed test so all 1% goes into the RIGHT or POSITIVE tail. That translates to z(2.325)=.4900.

Step 5: State the decision rule.

Reject the null if the TR >2.325, otherwise FTR.

Step 6: Perform necessary calculations on data and compute TR value.

$\sigma_x = \dfrac{s}{\sqrt{n}} = \dfrac{7}{\sqrt{49}} = \dfrac{7}{7} = 1$

TR = $\dfrac{\bar{x} - \mu_{Ho}}{\sigma_x} = \dfrac{103 - 100}{1} = \dfrac{3}{1} = 3$

Step 7: Compare TR value with the decision rule and make a statistical decision. (Write out decision in English! -- my addition)

+3 falls in the rejection region Therefore we REJECT NULL and CAN conclude that mean test scores in the population are higher than 100 with 99% confidence

P-value in plain English

p=100%-99.87%=0.13% or .0013 $p<\alpha$ therefore reject the null hypothesis

In plain English, you reject the null and conclude the mean test scores in the population are higher than 100 with only a 0.13% chance of error (or with 99.87% chance of being correct).

6 # of prior DUIs >0 WHEN α =.05?

s=3.564 n=28 $\bar{x}$= 1.96 μ_{Ho}=0 α =.05

Step 1: State the null and alternative hypothesis (H_0 and H_1).

H_0: $\mu \leq 0$
H_1: $\mu > 0$

Step 2: State level of significance or α "alpha."

For this example we'll use α =.05

Step 3: Determine the test distribution to use – z or t.

For this example, n<30 therefore we must assume that the population from which we sampled is NORMALLY DISTRIBUTED

Step 4 and 5

In this case, we have one tailed test so all 5% goes into the RIGHT or POSITIVE tail. df=n-1 or 28-1=27. t value for .05 in the tail is 1.703

Step 5: State the decision rule.

Reject the null if the TR>1.703, otherwise FTR.

Step 6: Perform necessary calculations on data and compute TR value.

$$\sigma_x = \frac{s}{\sqrt{n}} = \frac{3.564}{\sqrt{28}} = \frac{3.564}{5.29} = 0.674$$

$$\text{TR} = \frac{\bar{x} - \mu_{Ho}}{\sigma_x} = \frac{1.96 - 0}{.674} = \frac{1.96}{.674} = 2.9$$

Step 7: Compare TR value with the decision rule and make a statistical decision. (Write out decision in English! -- my addition)

+2.9 falls in the rejection region. Therefore we REJECT NULL and CAN conclude that mean number of prior DUI convictions for the population of BAC test refusers is greater than 0 with 95% confidence

P-value

We do not compute p-value by hand when we use t in this class

7: BAC higher than .08 WHEN α =.01?

s=0.063 n=475 $\bar{x}$=0.13502 μ_{Ho}=0.08 α =.01

Step 1: State the null and alternative hypothesis (H_0 and H_1).

H_0: $\mu \leq 0.08$
H_1: $\mu > 0.08$

Step 2: State level of significance or α "alpha."

For this example we'll use α =.01

Step 3: Determine the test distribution to use – z or t.

For this example, although the population parameters are unknown, we have a sample size bigger than 30 so we use z

Step 4 and 5

In this case, we have one tailed test so all 1% goes into the RIGHT or POSITIVE tail. That translates to z(2.325)=.4900.

Step 5: State the decision rule.

Reject the null if the TR >2.325, otherwise FTR.

Step 6: Perform necessary calculations on data and compute TR value.

$$\sigma_x = \frac{s}{\sqrt{n}} = \frac{.063}{\sqrt{475}} = \frac{.063}{21.8} = .002872$$

$$\text{TR} = \frac{\bar{x} - \mu_{Ho}}{\sigma_x} = \frac{.13502 - .08}{.002872} = \frac{.05502}{.002872} = 19.15$$

Step 7: Compare TR value with the decision rule and make a statistical decision. (Write out decision in English! -- my addition)

+19.15 falls in the rejection region Therefore we REJECT NULL and CAN conclude that mean BAC in the population is higher than 0.08 with at least 99% confidence.

P-value

Our z table does not go up to 19.15 so we cannot compute p …it is very small!

8: BAC higher than .10 WHEN α =.01?

s=0.063 n=475 $\bar{x}$=0.13502 μ_{Ho}=0.10 α =.01

Step 1: State the null and alternative hypothesis (H_0 and H_1).

H_0: $\mu \leq 0.10$

H_1: $\mu > 0.10$

Step 2: State level of significance or α "alpha."

For this example we'll use α =.01

Step 3: Determine the test distribution to use – z or t.

For this example, although the population parameters are unknown, we have a sample size bigger than 30 so we use z

Step 4 and 5

In this case, we have one tailed test so all 1% goes into the RIGHT or POSITIVE tail. That translates to z(2.325)=.4900.

Step 5: State the decision rule.

Reject the null if the TR >2.325, otherwise FTR.

Step 6: Perform necessary calculations on data and compute TR value.

$$\sigma_x = \frac{s}{\sqrt{n}} = \frac{.063}{\sqrt{475}} = \frac{.063}{21.8} = .002872$$

$$\text{TR} = \frac{\bar{x} - \mu_{Ho}}{\sigma_x} = \frac{.13502 - .1}{.002872} = \frac{.03502}{.002872} = 12.19$$

Step 7: Compare TR value with the decision rule and make a statistical decision. (Write out decision in English! -- my addition)

+12.19 falls in the rejection region Therefore we REJECT NULL and CAN conclude that mean BAC in the population is higher than 0.10 with at least 99% confidence

P-value
Our z table does not go up to 12.19 so we cannot compute p …it is very small!