Lecture 18b: Practice problems for Two-Sample Hypothesis Test of Means

Practice

Everything that appears in these lecture notes is fair game for the test. They are the best "study guide" I can provide. It is impossible to provide a "list" that is more comprehensive than the lecture notes above. However, here are a few additional practice exercises or practice concepts.

1. Do BAC test "refusers" have a different number of prior DUI arrests than BAC test "takers?"

I did a study in Hawaii on drunk driving and one idea was that people who refuse to take a Blood Alcohol Content Test (BAC test) are refusing because they have been arrested for DUI before and are trying to avoid "being caught" again. So we compared the mean number of prior DUI arrests between BAC test "refusers" and BAC test takers. FAKE DATA

μ_1 = BAC test refusers: $\bar{x}$ =1.96 prior DUI arrests with a s=10 arrests n=100

μ_2 = BAC test takers: $\bar{x}$ = .99 prior DUI arrests with a s= 10 arrests n=100

ASSUME $\sigma_1^2 \neq \sigma_2^2$ AND α=.05

Test the hypothesis that the mean number of prior DUI arrests are equal for the population of BAC test takers and the population of BAC test refusers. Or another way of saying it: do a hypothesis test to "prove" that the mean number of prior DUI arrests for the two populations are not equal.

ANSWER

1. **State the null and alternative hypothesis (H_0 and H_1).**

We do a "two sample two tailed test."

H_0: $\mu_1 = \mu_2$

H_1: $\mu_1 \neq \mu_2$

2. State level of significance or α "alpha."

For this example we'll use α =.05

3. Determine the test distribution to use – z or t.

For this example, although the population parameters are unknown, we have both sample sizes bigger than 30 so we use z distribution.

4 Define the rejection regions. And draw a picture!

In this case, we have two-tailed test so we split the 5% up – ½ in each tail. That translates to z(1.96)=.4750. Draw it out with both "acceptance regions" and "rejection regions."

5. State decision rule

Reject null if TR>1.96 or TR<-1.96 otherwise fail to reject the null

6. Perform necessary calculations on data and compute TR value.

In this case TR= $\frac{(\bar{x}_1 - \bar{x}_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{(1.96 - .99)}{\sqrt{\frac{100}{100} + \frac{100}{100}}} = \frac{.97}{\sqrt{1+1}} = \frac{.97}{\sqrt{2}} = \frac{.97}{1.414} = 0.686$

7. Compare TR value with the decision rule and make a statistical decision. Write out decision in English! -- my addition)

TR fall in fail to reject null region. There is insufficient evidence to reject the theory that the mean number of prior DUI arrests for the population of BAC test takers and for the population of BAC test refusers are equal.

p-value by hand: z(.68)=.2518. .5-.2518=.2482 and .2482 x 2 = .4964 or 49.64%. If you were to reject the null hypothesis you would have to accept a 49.64% chance of error. Or there is a 49.64% chance that mean number of prior DUI arrests are the same for BAC test refusers and BAC "takers."

2. Do BAC test "refusers" have more prior DUI arrests than BAC test "takers?"

I did a study in Hawaii on drunk driving and one idea was that people who refuse to take a Blood Alcohol Content Test (BAC test) are refusing because they have been arrested for DUI before and are trying to avoid "being caught" again. So we compared the mean number of prior DUI arrests between BAC test "refusers" and non-"refusers." FAKE DATA

μ_1 = BAC test refusers : $\bar{x}$ =1.96 prior DUI arrests with a s=10 arrests n=100

μ_2 = BAC test takers : $\bar{x}$ = .99 prior DUI arrests with a s= 10 arrests n=100

ASSUME $\sigma_1^2 \neq \sigma_2^2$ AND α=.05

Test the hypothesis that the mean number of prior DUI arrests for the population of BAC test refusers is equal to or less than for the population of BAC test takers. Or another way of saying it: do a hypothesis test to "prove" that the mean number of prior DUI arrests for the population of BAC test refusers is higher than for the population of BAC test takers.

ANSWER

1. State the null and alternative hypothesis (H_0 and H_1).

H_0: $\mu_1 \leq \mu_2$
H_1: $\mu_1 > \mu_2$

2. State level of significance or α "alpha."
For this example we'll use α =.05

3. Determine the test distribution to use – z or t.
For this example, although the population parameters are unknown, we have both sample sizes bigger than 30 so we use z distribution.

4. Define the rejection regions. And draw a picture!
In this case, we have a ONE TAILED test and all 5% goes in the RIGHT or POSITIVE tail. That translates to z(1.645)=.4500. Draw it out with both "acceptance regions" and "rejection regions."

5. State the decision rule

Reject null if TR>1.645 otherwise FAIL TO REJECT the null

6. Perform necessary calculations on data and compute TR value.

In this case TR= $\dfrac{(\bar{x}_1 - \bar{x}_2)}{\sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}} = \dfrac{(1.96 - .99)}{\sqrt{\dfrac{100}{100} + \dfrac{100}{100}}} = \dfrac{.97}{\sqrt{1+1}} = \dfrac{.97}{\sqrt{2}} = \dfrac{.97}{1.414} = 0.686$

7. Compare TR value with the decision rule and make a statistical decision. (Write out decision in English! -- my addition)

TR falls in the fail to reject the null region. There is insufficient evidence to reject the theory that the mean number of prior DUI arrests for the population of BAC test refusers is less than or equal to the mean number of prior DUI arrests for the population of BAC test takers.

p-value by hand: z(.68)=.2518. and .5-.2518=.2482 or 24.82% If you were to reject the null hypothesis you would have to accept a 24.82% chance of error. Or there is a 24.82% chance that mean number of prior DUI arrests for the population of BAC test refusers is equal to or less than the mean number of prior DUI arrests for the population of BAC "takers."

3. Do mean test scores differ before and after increased teacher training? α=.05

Pretend the Department of Education (DOE) claims that increasing teacher training has resulted in higher mean test scores for students. Well we could compare the mean test scores before the increased teacher training to the mean of test scores after the teacher training.

μ_1 = test score after teacher training: $\bar{x}$ =200 s=10 n=100

μ_2 = test score before teacher training : $\bar{x}$ = 197 s= 10 arrests n=100

ASSUME $\sigma_1^2 \neq \sigma_2^2$ AND α=.05

Test the hypothesis that the mean test scores after teacher training is equal to the mean test scores before teacher training. Or "prove" that the mean test scores before and after teacher training are different. Again inserting word "population" can be confusing as there is a population of all test scores before the training and a population of all test scores after the training.

ANSWER

1. **State the null and alternative hypothesis (H_0 and H_1).**

H_0: $\mu_1 = \mu_2$
H_1: $\mu_1 \neq \mu_2$

2. State level of significance or α "alpha."
For this example we'll use α =.05

3. Determine the test distribution to use – z or t.
For this example, although the population parameters are unknown, we have both sample sizes bigger than 30 so we use z distribution.

4 Define the rejection regions. And draw a picture!
In this case, we have two-tailed test so we split the 5% up – ½ in each tail. That translates to z(1.96)=.4750. Draw it out with both "acceptance regions" and "rejection regions."

5. State decision rule

Reject null if TR>1.96 or TR< -1.96 otherwise fail to reject the null

6. Perform necessary calculations on data and compute TR value.

In this case TR= $\frac{(\bar{x}_1 - \bar{x}_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{(200-197)}{\sqrt{\frac{100}{100}+\frac{100}{100}}} = \frac{3}{\sqrt{1+1}} = \frac{3}{\sqrt{2}} = \frac{3}{1.414} = 2.1$

7. Compare TR value with the decision rule and make a statistical decision. (Write out decision in English! -- my addition)

TR falls in the rejection region. Therefore reject null and conclude the alternative. In plain English the mean test scores of the population of students before teacher training are different than the mean test scores the population of students after teacher training. We are at least 95% confident of this decision.

p-value by hand: z(2.1)=.4821. .5-.4821= .0179 and .0179 x 2 = .0358 or 3.58%. You can conclude that the mean tests scores before and after teacher training are not equal (but you have to accept a 3.58% chance of error). Or there is a 3.58% chance that mean test scores before and after teacher training are equal.

4. Do mean test scores go up due to increased teacher training? α=.05

Pretend the Department of Education (DOE) claims that increasing teacher training has resulted in higher mean test scores for students. Well we could compare the mean test scores before the increased teacher training to the mean of test scores after the teacher training.

μ_1 = test score after teacher training: $\bar{x}$ =200 s=10 n=100

μ_2 = test score before teacher training : $\bar{x}$ = 197 s= 10 arrests n=100

ASSUME $\sigma_1^2 \neq \sigma_2^2$ AND α=.05

Test the hypothesis that the mean test scores after teacher training are LESS THAN OR EQUAL TO the mean test scores before teacher training. Or "prove" that the mean test scores after teacher training are higher than the mean test scores before teacher training. Again inserting word "population" can be confusing as there is a population of all test scores before the training and a population of all test scores after the training.

ANSWER

1. **State the null and alternative hypothesis (H_0 and H_1).**

H_0: $\mu_1 \leq \mu_2$

H_1: $\mu_1 > \mu_2$

2. State level of significance or α "alpha."

For this example we'll use α =.05

3. Determine the test distribution to use – z or t.

For this example, although the population parameters are unknown, we have both sample sizes bigger than 30 so we use z distribution.

4 Define the rejection regions. And draw a picture!

In this case, we have a ONE TAILED test and all 5% goes in the RIGHT or POSITIVE tail. That translates to z(1.645)=.4500. Draw it out with both "acceptance regions" and "rejection regions."

5. State decision rule

Reject null if TR>1.645 otherwise fail to reject the null

6. Perform necessary calculations on data and compute TR value.

In this case TR= $\dfrac{(\bar{x}_1 - \bar{x}_2)}{\sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}} = \dfrac{(1.96 - .99)}{\sqrt{\dfrac{100}{100} + \dfrac{100}{100}}} = \dfrac{3}{\sqrt{1+1}} = \dfrac{3}{\sqrt{2}} = \dfrac{3}{1.414} = 2.1$

7. Compare TR value with the decision rule and make a statistical decision. (Write out decision in English! -- my addition)

TR falls in rejection region. Conclude that the mean test scores of the population after teacher training are higher than the mean test scores of the population before teacher training. In other words, the teacher training increased student test scores. We are at least 95% confident of this decision.

p-value by hand: z(2.1)=.4821. .5-.4821= .0179 or 1.79%. You can conclude that the mean tests scores after teacher training are higher than before teacher training (but you have to accept a 1.79% chance of error). Or there is a 1.79% chance that the mean test scores after teacher training are equal to or less than the mean test scores before teacher training.

5. Do mean test scores go up due to increased teacher training? α=.01

Pretend the Department of Education (DOE) claims that increasing teacher training has resulted in higher mean test scores for students. Well we could compare the mean test scores before the increased teacher training to the mean of test scores after the teacher training.

μ_1 = test score after teacher training: $\bar{x}$ =200 s=10 n=100

μ_2 = test score before teacher training : $\bar{x}$ = 197 s= 10 arrests n=100

ASSUME $\sigma_1^2 \neq \sigma_2^2$ AND α=.01

Test the hypothesis that the mean test scores of the population after teacher training are LESS THAN OR EQUAL TO the mean test scores of the population before teacher training. Or "prove" that the mean test scores after teacher training are higher than before teacher training.

ANSWER

1. **State the null and alternative hypothesis (H_0 and H_1).**

H_0: $\mu_1 \leq \mu_2$
H_1: $\mu_1 > \mu_2$

2. State level of significance or α "alpha."

For this example we'll use α =.01

3. Determine the test distribution to use – z or t.

For this example, although the population parameters are unknown, we have both sample sizes bigger than 30 so we use z distribution.

4 Define the rejection regions. And draw a picture!

In this case, we have a ONE TAILED test and all 1% goes in the RIGHT or POSITIVE tail. That translates to z(2.235)=.4900. Draw it out with both "acceptance regions" and "rejection regions."

5. State decision rule

Reject null if TR>2.235 otherwise fail to reject the null

6. Perform necessary calculations on data and compute TR value.

In this case TR= $\dfrac{(\bar{x}_1 - \bar{x}_2)}{\sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}} = \dfrac{(1.96 - .99)}{\sqrt{\dfrac{100}{100} + \dfrac{100}{100}}} = \dfrac{3}{\sqrt{1+1}} = \dfrac{3}{\sqrt{2}} = \dfrac{3}{1.414} = 2.1$

7. Compare TR value with the decision rule and make a statistical decision. (Write out decision in English! -- my addition)

TR falls in fail to reject region. There is insufficient evidence to reject the theory that the mean test scores of the population after teacher training LESS THAN OR EQUAL TO the mean test scores of the population before teacher training.

p-value by hand: z(2.1)=.4821. .5-.4821= .0179 or 1.79% You can conclude that the mean tests scores after teacher training are higher than before teacher training (but you have to accept a 1.79% chance of error). Or there is a 1.79% chance that mean test scores after teacher training are equal to or less than mean test scores before teacher training.

6. Did mean "time waiting in line" at City Hall drop after implementation of program?

Lastly pretend the mayor instituted a program designed to lower the amount of time people wait in line to receive services at City Hall. We could compare the mean waiting time before implementation of this program to the mean waiting time taken from a sample after implementation of the program. The mean waiting time should be less after implementation of the program if it works. See lecture 18 but there is a population of mean waiting times before the program and a population of mean waiting times after the program.

μ_1 = mean waiting time BEFORE program: $\bar{x}$ =6 s=10 n=100

μ_2 = mean waiting time AFTER program: $\bar{x}$ = 4 s= 10 n=100

ASSUME $\sigma_1^2 \neq \sigma_2^2$ AND α=.01

Test the hypothesis that the mean waiting time for the population after implementation of program is LESS THAN OR EQUAL TO the mean waiting time for the population before implementation of the program. Or "prove" that the mean waiting time for the population after implementation of the program is greater than for the population before the program was started.

ANSWER

1. **State the null and alternative hypothesis (H_0 and H_1).**

H_0: $\mu_1 \leq \mu_2$
H_1: $\mu_1 > \mu_2$

2. State level of significance or α "alpha."
For this example we'll use α =.01

3. Determine the test distribution to use – z or t.
For this example, although the population parameters are unknown, we have both sample sizes bigger than 30 so we use z distribution.

4 Define the rejection regions. And draw a picture!
In this case, we have a ONE TAILED test and all 1% goes in the RIGHT or POSITIVE tail. That translates to z(2.235)=.4900. Draw it out with both "acceptance regions" and "rejection regions."

5. State decision rule

Reject null if TR>2.235 otherwise fail to reject the null

6. Perform necessary calculations on data and compute TR value.

In this case TR= $\frac{(\overline{x}_1 - \overline{x}_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{(6-4)}{\sqrt{\frac{100}{100} + \frac{100}{100}}} = \frac{2}{\sqrt{1+1}} = \frac{2}{\sqrt{2}} = \frac{2}{1.414} = 1.4$

7. Compare TR value with the decision rule and make a statistical decision. (Write out decision in English! -- my addition)

TR falls in the FAIL TO REJECT region. There is insufficient evidence to reject the theory that the mean waiting time for the population after implementation of the program is greater than or equal to the mean waiting time for the population before implementation of the program. In plain English it appears that the mayor's program did not lower mean waiting time in line for people.

p-value by hand: z(1.4)=.4192, that means p=.5 - 0.4192=0.0808 or 8.08%. That means if you were to reject the null and conclude that the mean waiting time is LOWER after implementation of the program you would have to accept an 8.08% chance of being wrong. Or there is an 8.08% chance that the null hypothesis is correct. There is an 8.08% chance that the theory that the mean waiting time after implementation of the program is greater than or equal to the mean waiting time before implementation of the program is correct. Again hard to insert the word "population" here.