Michael Hallstone, Ph.D.
hallston@hawaii.edu

Lecture 19: One Way ANOVA

Some Common Sense Assumptions for ANOVA:

1. The variable used is appropriate for a mean (interval/ratio level). **(Hint for exam: no student project should ever violate this nor have to assume it. Your data set will this sort of variable.)**

2. The data comes from a random sample**. (Hint for exam: all student projects violate this assumption.)**

3. The (test variable in each) populations under study is normally distributed. Now we have at least 3 populations. If we tested the mean age of 3 different ethnicities we would assume that age was normally distributed in each of the 3 ethnic populations. . **Hint for exam: you will make this assumption!**
4. The populations from which the samples are drawn from all have the same (unknown) variance. $\sigma^2_1 = \sigma^2_2 = \sigma^2_3 = \ldots\sigma$□□**(Hint for exam: you do not have to mention this assumption, most likely you are okay.)**
5. The samples are independent of each other. This is the same type of independence as for two sample hypothesis tests (but of course in ANOVA we have more than two samples). The two samples are independent. Definition of *"independence*" from the book says, "A sample selected from the first population is said to be an independent sample if it isn't related in some way to the data source found in the second population." The sixth edition defines *dependence* as, "If the same (or related) data sources are used to generate the data sets for each population, then the samples taken from each population are said to be dependent." An example of two dependent samples would be any "before and after" type of test or measure, or if we were looking at the average age between parents and children. By definition we know that a parent is older than their children. Also if we were looking at mean age between husbands and wives, typically (not always) husbands tend to be older than their wives. In these two cases the samples are dependent. **(Hint for exam: you do not have to mention this assumption, most likely you are okay.)**

Theory Behind one way ANOVA

Two sample hypothesis tests allowed us to check to see if two means were equal. But what do we do if we want to compare more than 2 means? ANOVA or analysis of variance allows us to check for equality of means for more than 2 samples (or groups).

We will use one-way ANOVA which only allows us to look at one variable at a time. There are ANOVA tests (two-way, etc.) that allow us to examine multiple variables, but we will not cover them in this course. Just know that they exist and you may cover them in higher statistics.

Essentially the 7 steps to hypothesis testing are used in ANOVA with a few differences. The "test ratio or TR" formula is computed as a ratio or fraction of

$$\text{ANOVA “TR”} = \frac{\text{the average variation between groups}}{\text{average variance within groups}}$$

So there are two ways for any fraction to produce a large number – a big top number or a small bottom number.

Recall to get the number of fraction you divide the top number by the bottom number: e.g. ½ = 1 divided by 2 = 0.5.

When the top number in a fraction is small that means a small number overall:

10/10= 1

5/10 = 0.5

1/10 = 0.1

In the fractions above, we kept the bottom number the same and made the top number smaller. Notice how the "overall number" of the fraction got smaller? The same happens if we keep the top number the same but make the bottom number smaller – the number the fraction represents gets bigger.

10/10=1 10/5=2 10/2=5 10/1=10.

Well back to our TR test. If the average variation between groups is small (the top number in TR fraction) then we will have small TR. As in all the other tests a small TR means likely that we will Fail to Reject the Null (FTR). And if the variation with in the groups is smaller it also make the TR fraction a bigger number.

Another way to look at between groups variation and its relationship to the TR ratio: sample mean is an unbiased estimator of population mean, therefore sample mean *should* be close to population mean, implying a very small variation between groups. If they all come from the same population, then there should be very small variation among the means. Small variation amongst the means leads to FTR.

For example look at the following means:

Example 1		**Example 2**	
Front	40	**front**	22
Middle	41	**middle**	40
Back	42	**back**	60
mean	**41**		**41**

Although both groups have essentially the same mean, the first set of numbers has very little variation between the 3 groups and the second one has relatively large variation. If the means in Example 1come from the same population, then the variance of means between groups should be small -- implying a small numerator (top number in the TR fraction) and FTR. In example 2, the means probably come from different populations as the variance of means between the groups is larger – implying a large numerator and a “reject the null” decision in step 7.

The denominator of the TR ratio is computed differently and we will examine that later in the lecture.

Example of One-way ANOVA where we reject the null hypothesis

See the bottom of [lecture 19c: SPSS output (19c_SPSS.pdf)] for an example of ANOVA where you fail to reject the null hypothesis.

In this example the actual #'s are from the problem done in the book Statistics: A First Course (section 11.2 in the fifth edition pg. 423-426 and section 10.2 in the sixth edition pp448-451). The numbers are from this problem, but I changed the story for the numbers to make it easier to understand.

Grocery stores put milk in the back of the store. If you come in just to buy milk, you will have to walk through the whole store to get to the milk and then you might see something else you need, thereby increasing total sales volume.

Pretend the C&C of Honolulu is going to use this theory and put a small retail store in the front of its Satellite City Hall location in Ala Moana Shopping Center. The idea is that if you come in to license your car, but you have to walk through a small retail store before you get to the counter and that store has cool little things to buy (i.e. lifeguard coffee cups, firefighter t-shirt, etc) you will buy something on the way to do your official county business. So you came to the city hall to license your car, but you end up buying some of their retail merchandise. In this way the C&C of Honolulu might make more money!

So pretend the manager in the county want to do a pilot study to see if the location of the retail store inside of the city hall actually affect the sales volume of the retail items. They expand this program in its other locations, so it will do a pilot study an see if sales are affected by where the retail store is located – in the front, middle, or back of the building.

So retail stores are placed in 3 different locations in 3 different stores and sales volume is monitored. Each store manager randomly samples total sales for 6 days (n=6).

Total Sales (in thousands) by location of retail section in Satellite City Hall

Day	front	middle	back
x1	45	55	54
x2	56	50	61
x3	47	53	54
x4	51	59	58
x5	50	58	52
x6	45	49	51

Test the theory or hypothesis that the mean sales are equal in the three populations that correspond to the three store locations (front, middle, and back). Conversely do an ANOVA test to prove that not all three populations that correspond to the three store locations (front, middle, and back) have equal mean sales.

another version of the plain English

Test the hypothesis or theory that the mean sales are equal in the three populations that correspond to the three store locations. Conversely try to prove that the mean sales are not equal in the three populations that correspond to the three store locations.

1. **State the null and alternative hypothesis (H_0 and H_1).**

   H_0: $\mu_F = \mu_M = \mu_B$

   H_1: not all population means are equal

2. **State level of significance or α "alpha."**

   For this example we'll use alpha =.05

3. **Determine the test distribution to use – ANOVA tests use F distribution.**

4. **Define the rejection regions. And draw a picture! (You need the f table for this part)**

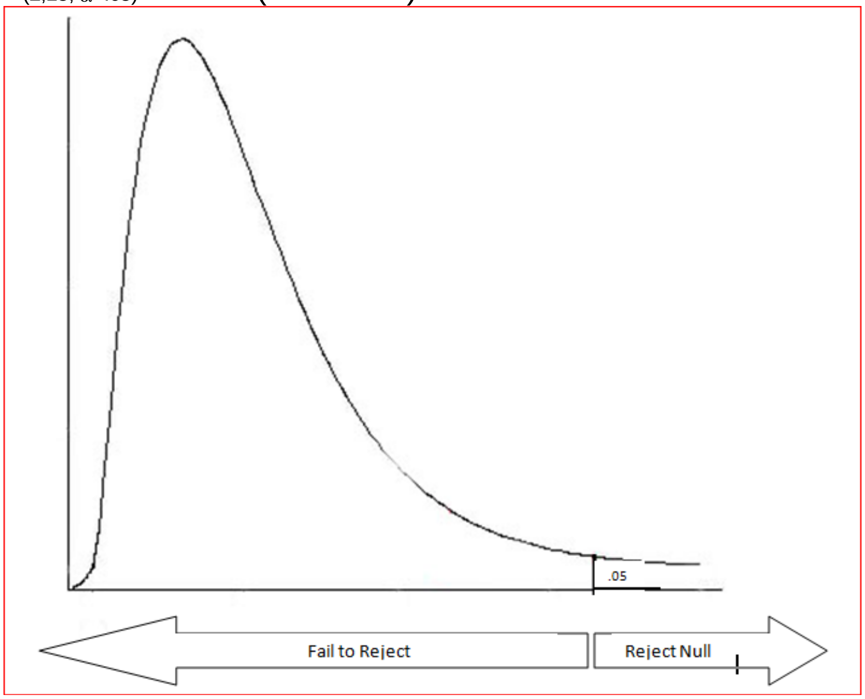
$df_{numerator}$=k-1 $df_{denominator}$=T-k

k= # of samples T=total # of items in all samples

for "samples" think number of groups or categories the I/R variable is "chopped up" into. Our I/R variable ($milk sales in $1,000's) is chopped up into 3 categories (front, middle, back) by an ordinal level variable.

in this case $df_{numerator}$=3-1 =2 & $df_{denominator}$=18-3=15

$F_{(2,15,\ \alpha=.05)}$=3.68 (draw it out)



3.68

Digital artwork courtesy of Janet Howze. Thanks Janet!

5. **State the decision rule.**

Reject the null if the TR >3.68, otherwise FTR.

6. **Perform necessary calculations on data and compute TR value.**

In this case

$$TR=\frac{\hat{s}^2 between}{\hat{s}^2 within}$$

$\sigma^2_{between}$

Day	data (x)	Day	data (x)	Day	data (x)
	Front		Middle		Back
x1	45	x1	55	x1	54
x2	56	x2	50	x2	61
x3	47	x3	53	x3	54
x4	51	x4	59	x4	58
x5	50	x5	58	x5	52
x6	45	x6	49	x6	51
Sum	**294**	**Sum**	**324**	**Sum**	**330**
mean	**49**	**Mean**	**54**	**mean**	**55**

grand mean=52.6 (49+54+55)/3=52.6

$$\sigma^2_{between} = \frac{n_1(\text{x-bar}_1\text{-grand mean})^2 + n_2(\text{x-bar}_2\text{-grand mean})^2 + n_3(\text{x-bar}_3\text{-grand mean})^2}{k-1}$$

$$= \frac{6(49-52.6)^2 + 6(54-52.6)^2 + 6(55-52.6)^2}{3-1}$$

$$\frac{6(-3.6)^2 + 6(1.4)^2 + 6(2.4)^2}{2} = \frac{6(12.96) + 6(1.96) + 6(5.76)}{2} = \frac{77.76 + 11.76 + 34.56}{2} = \frac{124.08}{2}$$

$$= \frac{124}{2} = 62$$

σ^2_{within}

"Front" store -- milk case in front of store						"Middle" store -- milk case in middle of store				
day	data (x)	mean	x - mean	(x-mean)2		day	data (x)	mean	x - mean	(x-mean)2
x1	45	49	-4	16		x1	55	54	1	1
x2	56	49	7	49		x2	50	54	-4	16
x3	47	49	-2	4		x3	53	54	-1	1
x4	51	49	2	4		x4	59	54	5	25
x5	50	49	1	1		x5	58	54	4	16
x6	45	49	-4	16		x6	49	54	-5	25
sum	294		0	90		sum	324	54	0	84
mean	49					mean	54			

"Back" store -- milk case in back of store				
day	data (x)	mean	x - mean	(x-mean)2
x1	54	55	-1	1
x2	61	55	6	36
x3	54	55	-1	1
x4	58	55	3	9

x5	52	55	-3	9
x6	51	55	-4	16
sum	330	55	0	72
mean	55			

$$\sigma^2_{within} = \frac{\Sigma d^2_1 + \Sigma d^2_2 + \Sigma d^2_3 + \Sigma d^2_k}{T - k}$$

Σd^2_1= sum of squared differences for first sample: $\sum (x-\bar{x})^2$

Σd^2_2= sum of squared differences for second sample: $\sum (x-\bar{x})^2$ etc. etc etc.

T= total # of all items in the all samples ($n_1 + n_2 + n_3 + \ldots + n_k$)

k= number of samples (think "groups" or "categories")

$$\sigma^2_{within} = \frac{90 + 84 + 72}{18-3} = \frac{246}{15} = 16.4$$

TR Calculation

$TR = \frac{\sigma^2_{between}}{\sigma^2_{within}}$ (Carrot hats over the σ's!!)

$= \frac{62.0}{16.4} = 3.78$

7. **Compare TR value with the decision rule and make a statistical decision. (Write out decision in English! -- my addition)**

Comparing our TR value to the critical value stated in step #5, we see that TR> 3.68. Therefore, reject the null and conclude with at least 95% confidence that at least one of the populations that correspond to the three different City Hall retail store locations is likely to have greater or fewer mean sales than the other. Additional research is needed to define the nature of this relationship between total sales and location of retail store placement.

What is the wording when we fail to reject the null hypothesis?

When we fail to reject the null hypothesis we say "Insufficient evidence to reject theory that __________ " [insert Ho in plain English.]

You do not “conclude” null, so much as you can only say “insufficient evidence to reject theory in null” which relates to the way science progresses. Until there is evidence to reject the theory the theory stands. We are not 95% confident or anything like that.

See the bottom of lecture 19c: SPSS output (19c_SPSS.pdf) for an example of ANOVA where you fail to reject the null hypothesis

SPSS output below…

The Beauty of SPSS

(See lecture 19c: SPSS output (19c_SPSS.pdf) for full instructions.)

This is all so much easier with the computer. The computer did in fractions of a second what took us a whole lot longer by hand.

I just created two variables: ***sales*** = *sales in thousands of $* (continuous ratio variable) and ***location***= *location of retail store* (ordinal discrete variable where 1= front, 2= middle, and 3= back).

Here is the data:

	sales	location
1	45.00	1.00
2	56.00	1.00
3	47.00	1.00
4	51.00	1.00
5	50.00	1.00
6	45.00	1.00
7	55.00	2.00
8	50.00	2.00
9	53.00	2.00
10	59.00	2.00
11	58.00	2.00
12	49.00	2.00
13	54.00	3.00
14	61.00	3.00
15	54.00	3.00
16	58.00	3.00
17	52.00	3.00
18	51.00	3.00

Here is the output:

ANOVA

sales in thousands$

	Sum of Squares	df	Mean Square	F	Sig.
Between Groups	124.000	2	62.000	3.780	.047
Within Groups	246.000	15	16.400		
Total	370.000	17			

below is additional output I requested from SPSS while doing ANOVA

Descriptives

sales in thousands$

	N	Mean	Std. Deviation	Std. Error	95% Confidence Interval for Mean		Minimum	Maximum
					Lower Bound	Upper Bound		
front	6	49.0000	4.24264	1.73205	44.5476	53.4524	45.00	56.00
middle	6	54.0000	4.09878	1.67332	49.6986	58.3014	49.00	59.00
back	6	55.0000	3.79473	1.54919	51.0177	58.9823	51.00	61.00
Total	18	52.6667	4.66527	1.09961	50.3467	54.9866	45.00	61.00

F Table for .05

Critical Values of the F-Distribution: $\alpha = 0.05$

Denom. d.f.	Numerator Degrees of Freedom 1	2	3	4	5	6	7	8	9	10
1	161.448	199.500	215.707	224.583	230.162	233.986	236.768	238.883	240.543	241.882
2	18.513	19.000	19.164	19.247	19.296	19.330	19.353	19.371	19.385	19.396
3	10.128	9.552	9.277	9.117	9.013	8.941	8.887	8.845	8.812	8.786
4	7.709	6.944	6.591	6.388	6.256	6.163	6.094	6.041	5.999	5.964
5	6.608	5.786	5.409	5.192	5.050	4.950	4.876	4.818	4.772	4.735
6	5.987	5.143	4.757	4.534	4.387	4.284	4.207	4.147	4.099	4.060
7	5.591	4.737	4.347	4.120	3.972	3.866	3.787	3.726	3.677	3.637
8	5.318	4.459	4.066	3.838	3.687	3.581	3.500	3.438	3.388	3.347
9	5.117	4.256	3.863	3.633	3.482	3.374	3.293	3.230	3.179	3.137
10	4.965	4.103	3.708	3.478	3.326	3.217	3.135	3.072	3.020	2.978
11	4.844	3.982	3.587	3.357	3.204	3.095	3.012	2.948	2.896	2.854
12	4.747	3.885	3.490	3.259	3.106	2.996	2.913	2.849	2.796	2.753
13	4.667	3.806	3.411	3.179	3.025	2.915	2.832	2.767	2.714	2.671
14	4.600	3.739	3.344	3.112	2.958	2.848	2.764	2.699	2.646	2.602
15	4.543	3.682	3.287	3.056	2.901	2.790	2.707	2.641	2.588	2.544
16	4.494	3.634	3.239	3.007	2.852	2.741	2.657	2.591	2.538	2.494
17	4.451	3.592	3.197	2.965	2.810	2.699	2.614	2.548	2.494	2.450
18	4.414	3.555	3.160	2.928	2.773	2.661	2.577	2.510	2.456	2.412
19	4.381	3.522	3.127	2.895	2.740	2.628	2.544	2.477	2.423	2.378
20	4.351	3.493	3.098	2.866	2.711	2.599	2.514	2.447	2.393	2.348
21	4.325	3.467	3.072	2.840	2.685	2.573	2.488	2.420	2.366	2.321
22	4.301	3.443	3.049	2.817	2.661	2.549	2.464	2.397	2.342	2.297
23	4.279	3.422	3.028	2.796	2.640	2.528	2.442	2.375	2.320	2.275
24	4.260	3.403	3.009	2.776	2.621	2.508	2.423	2.355	2.300	2.255
25	4.242	3.385	2.991	2.759	2.603	2.490	2.405	2.337	2.282	2.236
26	4.225	3.369	2.975	2.743	2.587	2.474	2.388	2.321	2.265	2.220
27	4.210	3.354	2.960	2.728	2.572	2.459	2.373	2.305	2.250	2.204
28	4.196	3.340	2.947	2.714	2.558	2.445	2.359	2.291	2.236	2.190
29	4.183	3.328	2.934	2.701	2.545	2.432	2.346	2.278	2.223	2.177
30	4.171	3.316	2.922	2.690	2.534	2.421	2.334	2.266	2.211	2.165
31	4.160	3.305	2.911	2.679	2.523	2.409	2.323	2.255	2.199	2.153
32	4.149	3.295	2.901	2.668	2.512	2.399	2.313	2.244	2.189	2.142
33	4.139	3.285	2.892	2.659	2.503	2.389	2.303	2.235	2.179	2.133
34	4.130	3.276	2.883	2.650	2.494	2.380	2.294	2.225	2.170	2.123
35	4.121	3.267	2.874	2.641	2.485	2.372	2.285	2.217	2.161	2.114
36	4.113	3.259	2.866	2.634	2.477	2.364	2.277	2.209	2.153	2.106
37	4.105	3.252	2.859	2.626	2.470	2.356	2.270	2.201	2.145	2.098
38	4.098	3.245	2.852	2.619	2.463	2.349	2.262	2.194	2.138	2.091
39	4.091	3.238	2.845	2.612	2.456	2.342	2.255	2.187	2.131	2.084
40	4.085	3.232	2.839	2.606	2.449	2.336	2.249	2.180	2.124	2.077
41	4.079	3.226	2.833	2.600	2.443	2.330	2.243	2.174	2.118	2.071
42	4.073	3.220	2.827	2.594	2.438	2.324	2.237	2.168	2.112	2.065
43	4.067	3.214	2.822	2.589	2.432	2.318	2.232	2.163	2.106	2.059
44	4.062	3.209	2.816	2.584	2.427	2.313	2.226	2.157	2.101	2.054
45	4.057	3.204	2.812	2.579	2.422	2.308	2.221	2.152	2.096	2.049
46	4.052	3.200	2.807	2.574	2.417	2.304	2.216	2.147	2.091	2.044
47	4.047	3.195	2.802	2.570	2.413	2.299	2.212	2.143	2.086	2.039
48	4.043	3.191	2.798	2.565	2.409	2.295	2.207	2.138	2.082	2.035
49	4.038	3.187	2.794	2.561	2.404	2.290	2.203	2.134	2.077	2.030
50	4.034	3.183	2.790	2.557	2.400	2.286	2.199	2.130	2.073	2.026
60	4.001	3.150	2.758	2.525	2.368	2.254	2.167	2.097	2.040	1.993