Chapter 3: Motion in a Plane
Goals for Chapter 3

• To study and calculate position, velocity, and acceleration vectors in 2D
• To frame two-dimensional motion as it occurs in the motion of projectiles.
• To use the equations of motion for constant acceleration to solve for unknown quantities for an object moving under constant acceleration in 2D
• To study the relative velocity of an object for observers in different frames of reference in 2D
Velocity in a Plane

- Vectors in terms of Cartesian $x$- and $y$- coordinates may now also be expressed in terms of magnitude and angle.

\[ r = |\vec{r}| = \sqrt{x^2 + y^2} \]
Velocity in a Plane

- From the graphs, we see both average and instantaneous velocity vectors.

Average velocity of a particle over displacement $\Delta \vec{r}$

$$\vec{v}_{av} = \frac{\Delta \vec{r}}{\Delta t}$$

Instantaneous velocity of a particle at point $\vec{r}$

$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t}$$

The instantaneous velocity vector $\vec{v}$ is always tangent to the x-y path.

$v_x$ and $v_y$ are the x and y components of $\vec{v}$. 
The Motion of a Model Car – Example 3.1

- See the worked example on page 67.

\[ \vec{v}_{av} \text{ points from } P_1 \text{ toward } P_2. \text{ It doesn’t matter how long you make it; its magnitude will be found mathematically.} \]

\[ \Delta y = 4.0 \text{ m} \]
\[ v_{av,y} = \frac{\Delta y}{\Delta t} \]
\[ t_1 = 2.0 \text{ s} \]

\[ \Delta x = 3.0 \text{ m} \]
\[ v_{av,x} = \frac{\Delta x}{\Delta t} \]

\[ t_2 = 2.5 \text{ s} \]
Accelerations in a Plane

- Acceleration must now be considered during change in magnitude AND/OR change in direction.

\[
\begin{align*}
\bar{a} &= \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t} \\
\bar{a}_{av} &= \frac{\Delta \vec{v}}{\Delta t}
\end{align*}
\]

Average acceleration of a particle over displacement \( \Delta \vec{r} \)

Instantaneous acceleration of particle at point \( \vec{r} \)

\[
\bar{a} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t}
\]
Accelerations in a Plane

- Acceleration must now be considered during change in magnitude AND/OR change in direction.
The Model Car Revisited – Example 3.2

• See the worked example on page 69.
Projectile Motion

- Determined by the initial velocity, gravity, and air resistance.
- Footballs, baseballs … any projectile will follow this **parabolic trajectory** in the x-y plane.

  - A projectile moves in a vertical plane that contains the initial velocity vector $\vec{v}_0$.
  - Its trajectory depends only on $\vec{v}_0$ and on the acceleration due to gravity.

\[ a_y = -g \]
The Independence of $x$- and $y$-Motion – Figure 3.9

- Notice how the vertical motion under free fall spaces out exactly as the vertical motion under projectile motion.
- We can treat the $x$- and $y$- coordinates separately!
• Notice the vertical velocity component as the projectile changes horizontal position.

\[ a_x = 0 \quad \& \quad a_y = -g \]

\[ g = +9.8 \, \text{m/s}^2 \quad \Rightarrow \text{magnitude of acceleration due to gravity} \]
**Constant Acceleration: Projectile Motion**

- We can treat the $x$- and $y$- coordinates separately!
- Toss something in the air: $a_x = 0$ and $a_y = -g$

<table>
<thead>
<tr>
<th>$x$-direction motion</th>
<th>$y$-direction motion</th>
</tr>
</thead>
<tbody>
<tr>
<td>$v_x = v_{ox}$</td>
<td>$v_y = v_{oy} - gt$</td>
</tr>
<tr>
<td>$x = v_x t$</td>
<td>$y = \frac{1}{2} (v_y + v_{oy}) t$</td>
</tr>
<tr>
<td>$x = v_{ox} t$</td>
<td>$y = v_{oy} t - \frac{1}{2} gt^2$</td>
</tr>
<tr>
<td>$v_x^2 = v_{ox}^2$</td>
<td>$v_y^2 = v_{oy}^2 - 2gy$</td>
</tr>
</tbody>
</table>
Don't forget:
Initial velocity is a 2D vector.

**Vector.** \( \vec{v}_0 = \vec{v}_{0x} + \vec{v}_{0y} \)

**Components.** \[
\begin{align*}
v_{oy} &= v_0 \sin \theta_0 \\
v_{ox} &= v_0 \cos \theta_0
\end{align*}
\]

**Initial speed.** \( v_0 = \sqrt{v_{ox}^2 + v_{oy}^2} \)

**Launch angle.** \( \tan \theta_0 = \frac{v_{oy}}{v_{ox}} \)
The Paintball Gun – Example 3.3

- See the worked example on page 74.
- Full consideration given to the motion after the dye-filled ball is fired.
A Home-Run Hit – Example 3.4

- See the fully worked example on pages 75–76.
- Details examined for the flight of a baseball hit from home plate toward a fence hundreds of feet away.
Vertical/Horizontal Displacement – Example 3.6

• After a given horizontal displacement, a projectile will have a vertical position.
• Sports provide a large number of excellent examples.
• For a field goal, see worked example on page 78.
Firing at a More Complex Target – Example 3.7

• A moving target presents a real-life scenario.
• It is possible to solve a falling body as the target. This problem is a "classic" on standardized exams.
• See the worked examples on pages 78–80.
Circular Motion as a Special Application – Figure 3.20

- Two dimensional motion in a plane takes on unique features when it is confined to a circle.
- The acceleration is centripetal ("center seeking").
- The velocity vector retains the same magnitude, it changes direction.

(a) Car speeding up along a circular path
(b) Car slowing down along a circular path
(c) Uniform circular motion: Constant speed along a circular path
It Is Possible to Solve for the Velocity – Figure 3.21

- The problem may be treated by extracting a small portion of the motion such that the $\Delta \vec{v}$ arc may be approximated as a straight line.

- Acceleration in a uniform circular motion is radial, which has magnitude:

\[
\alpha_{rad} = \frac{v^2}{r}
\]
An Example You Can Go Try Right Now – Example 3.8

- Refer to the worked example on page 82.

\[ v = 45 \text{ m/s} \]

\[ a_{\text{rad}} = 9.8 \text{ m/s}^2 \]

\[ R = ? \]
A Problem to Try on Your Next Vacation – Example 3.9

• Uniform circular motion applied to a daring carnival ride. [1 period = time for one revolution]

• Referred to the worked example on page 83.

\[ v = \frac{2\pi R}{T} \quad T = \text{period} \]

\[ a_{rad} = \frac{v^2}{R} \]
Relative Velocity: A Matter of Perspective – Figure 3.26

- Velocities can carry multiple values depending on the position and motion of the object and the observer.

\[ \vec{v}_{W/C} = \vec{v}_{W/T} + \vec{v}_{T/C} \]
An Airplane in a Crosswind – Example 3.10

- A solved application of relative motion.

\[ \vec{v}_{P/E} = \vec{v}_{P/A} + \vec{v}_{A/E} \]

- This is just velocity vector addition.