Chapter 6: Circular Motion and Gravitation
Goals for Chapter 6

• To understand the dynamics of circular motion.
• To study the unique application of circular motion as it applies to Newton's law of gravitation.
• To examine the idea of weight and relate it to mass and Newton's law of gravitation.
• To study the motion of objects in orbit as a special application of Newton's law of gravitation.
In Section 3.4

- We studied the kinematics of circular motion.
  - Centripetal acceleration
  - Changing velocity vector
  - Uniform circular motion
- We acquire new terminology.
  - Radian
  - Period \((T)\)
  - Frequency \((f)\)
Velocity Changing from the Influence of $a_{\text{rad}}$ – Figure 6.1

- A review of the relationship between $\nu$ and $a_{\text{rad}}$.
- The velocity changes direction, not magnitude.
- The magnitude of the centripetal acceleration is:
  \[ a_{\text{rad}} = \frac{\nu^2}{R} \]
- In terms of the speed and period (time to make one complete revolution)
  \[ \nu = \frac{2\pi R}{T} \implies a_{\text{rad}} = \frac{4\pi^2 R}{T^2} \]
Details of Uniform Circular Motion

• An object moves in a circle because of a centripetal force.
• Notice how \( \nu \) becomes linear when \( F_{\text{rad}} \) vanishes.

\[
F_{\text{net}} = F_{\text{rad}} = m \frac{\nu^2}{R}
\]
Model Airplane on a String – Example 6.1

- See the worked example on page 155.

\[ \mathbf{F}_x = m a_{\text{rad}}, \quad \mathbf{F}_T = \frac{m \mathbf{v}^2}{R} \]

\[ \sum F_y = 0, \quad F_{\text{lift}} + (-mg) = 0 \]
A Tether Ball Problem – Example 6.2

- Refer to the worked example on page 156.

\begin{align*}
\nu &= \frac{2\pi R}{T} \\
R &= L \sin \beta \\
\sum F_x &= ma_{\text{rad}}, \\
\sum F_y &= 0, \\
F_T \sin \beta &= m \frac{\nu^2}{R} = m \frac{4\pi^2}{T^2} \\
F_T \cos \beta + (-mg) &= 0
\end{align*}
Rounding a Flat Curve – Example 6.3

- The centripetal force coming only from static tire friction

$$\sum F_x = ma_{rad}, \quad f_s = m \frac{v^2}{R}$$
$$\sum F_y = 0, \quad n + (-mg) = 0$$

with $f_{s,\text{max}} = \mu_s n = \mu_s (mg)$

$$\Rightarrow \quad v_{\text{max}} = \sqrt{\mu_s g R}$$
Rounding a Banked Curve – Example 6.4

• The centripetal force comes only from a component of normal force

\[ \sum F_x = m a_{\text{rad}}, \quad n \sin \beta = m \frac{\nu^2}{R} \]

\[ \sum F_y = 0, \quad n \cos \beta + (-mg) = 0 \]

\[ \tan \beta = \frac{\nu^2}{gR} \]
Motion in a Vertical Circle

- Dynamics of a Ferris wheel – Example 6.5

(a) Sketch of two positions  (b) Free-body diagram for passenger at top  (c) Free-body diagram for passenger at bottom

- Normal force is position dependent \( (n_{bottom} > n_{top}) \)
Newton's Law of Gravitation – Figure 6.12

- Always attractive.
- Directly proportional to the masses involved.
- Inversely proportional to the square of the separation between the masses.
- Magnitude of force is given by:

\[ F_g = G \frac{m_1 m_2}{r^2} \]

- \( G \) is gravitational constant:

\[ G = 6.674 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2 \]
The Gravitational Force Calculated – Example 6.6

- Use Newton's law of universal gravitation with the specific masses and separation.
- Refer to the worked example on page 172.
This May be Done in a Lab – Cavendish Experiment (1798)

• The slight attraction of the masses causes a nearly imperceptible rotation of the string supporting the masses connected to the mirror. → use this to calculate $G$.

• Use of the laser allows a point many meters away to move through measurable distances as the angle allows the initial and final positions to diverge.

1. Gravitation pulls the small masses toward the large masses, causing the vertical quartz fiber to twist.

   The small balls reach a new equilibrium position when the elastic force exerted by the twisted quartz fiber balances the gravitational force between the masses.

2. The deflection of the laser beam indicates how far the fiber has twisted. Once the instrument is calibrated, this result gives a value for $G$. 

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Weight

- The weight of an object is the net gravitational force exerted on the object by all other objects in the universe.
- Weight of an object near the surface of the earth is:

\[ m_1 g = w = F_{g, \text{earth surface}} = G \frac{m_1 m_E}{R_E^2} \]

- With this we find that the acceleration due to gravity near the earth's surface is:

\[ g = G \frac{m_E}{R_E^2} = 9.8 \text{ m/s}^2 \text{ at surface of Earth} \]
Even Within the Earth Itself, Gravity Varies – Figure 6.16

- Distances from the center of rotation and different densities allow for interesting increase in $F_g$. 

![Diagram showing the Earth's internal structure with distances and density variations.](image)
Gravitational Force Falls off Quickly – Figure 6.15

- The gravitational force is proportional to $1/r^2$, and thus the weight of an object decreases inversely with the square of the distance from the earth's center (not distance from the surface of the earth).

$$w = \frac{Gm_Em}{r^2}$$

where

- $w$ = astronaut’s weight
- $r$ = astronaut’s distance from the center of the earth
- $R_E$ = astronaut’s distance from the surface of the earth

The figure illustrates the decrease in weight with increasing distance from the earth's center.
Gravitation Applies Elsewhere – Figure 6.17

- Mars
- See the worked example on pages 166–167.
Satellite Motion: What Happens When Velocity Rises?

- Eventually, $F_g$ balances and you have orbit.
- When $v$ is large enough, you achieve escape velocity.
- An orbit is not fundamentally different from familiar trajectories on earth. If you launch it slowly, it falls back. If you launch it fast enough, the earth curves away from it as it falls, and it goes into orbit.
Circular Satellite Orbit

- If a satellite is in a perfect circular orbit with speed $v_{\text{orbit}}$, the gravitational force provides the centripetal force needed to keep it moving in a circular path.

\[
\frac{G m_{\text{sat}} m_E}{r^2} = F_g = F_{\text{rad}} = m \frac{v^2}{R} \implies v_{\text{orbit}} = \sqrt{\frac{G m_E}{r}}
\]
Calculations of Satellite Motion – Example 6.10

- Work on an example of a relay designed to stay in orbit permanently.
- See the worked example on page 169.
If an Object is Massive, Even Photons Cannot Escape

• A "black hole" is a collapsed sun of immense density such that a tiny radius contains all the former mass of a star.

• The radius to prevent light from escaping is termed the "Schwarzschild Radius."

• The edge of this radius has even entered pop culture in films. This radius for light is called the "event horizon."