Chapter 11: Elasticity and Periodic Motion
Goals for Chapter 11

• To study stress, strain, and elastic deformation.
• To define elasticity and plasticity.
• To follow periodic motion to a study of simple harmonic motion.
• To solve equations of simple harmonic motion.
• To use the pendulum as a prototypical system undergoing simple harmonic motion.
• To study how oscillations may be damped or driven.
Stresses We Will Consider – Figure 11.1

- Stress is formally defined as force per unit area.
- Units would therefore be \( \text{N/m}^2 \), also known as the pascal (Pa).
- PSI, or \( \text{lb/in}^2 \), are the British equivalent.
- We will study four types of stress.
• Just to have an order of magnitude in mind while we read, a steel cable can withstand stresses on the order of several hundred million Pa.

We assume that the tension force is distributed evenly over any cross section through the bar.
As well as thinking of hanging heavy objects from cables we will consider the opposite motion, compression.

Consider a multiple story building and the forces upper floors exert on lower ones.

(a) A bar in compression

(b) Force on a cross section through the bar
If we return to the steel cable example, we could ask ourselves, "How much will the steel stretch under a load?"

Strain, then, is $\Delta l/l_0$, the unitless change in length divided by the original length.

Tensile stress $= \frac{F_{\perp}}{A}$

Tensile strain $= \frac{\Delta l}{l_0}$
The Relationship of Stress to Strain – Example 11.1

• Refer to Figure 11.6.
• We find stress proportional to strain.
• An equality may be constructed if we multiply strain by an elastic modulus.
• Follow the example on page 324.
Extend Our Thoughts to 2-D – Figure 11.9 and 11.10

- Sheer stress works in analogous fashion over two dimensions.
- Imagine a stack of paper that you slide unevenly.

A block subjected to shear stress from two directions

Shear stress $= \frac{F}{A}$
Shear strain $= \frac{x}{h}$
Earthquakes as Sheer Stress – Example 11.3

• We can consider plate movement using sheer stress.
• See Figure 11.11 and the worked example on page 339.
Stress and Strain in 3-D – Figures 11.7 and 11.8

• Our reasoning may be extended to volumes.

The force applied to the piston is distributed over the surface of the object.

At each point on the object, the force applied by the fluid is perpendicular to the object’s surface.

Initial volume $V_0$ at initial pressure $p_0$

Final volume $V$ at pressure $p = p_0 + \Delta p$

Volume stress $= \Delta p$

Volume strain $= \frac{\Delta V}{V_0}$
Elasticity, Plasticity, and Fracture – Figure 11.12

- As material encounters stress, initial effects are reproducible and reversible. These are elastic.
- As stress continues to rise, the effects are unpredictable and irreversible.
- If stress still continues to rise, fracture may occur.
Elastic Situations Yield Simple Harmonic Motion

- Shown here in Figure 11.14.
Pause and Consider Our Terminology – Figure 11.15

- Oscillation
- Restoring force
- SHM
- Amplitude ($A$) … in meters
- Cycle
- Period ($T$) … in seconds
- Frequency ($f$) … in 1/s or Hz
- Angular frequency ($\omega$) … in rad/s
Energy Changes as the Oscillator Moves – Figure 11.17

- Conserved in the absence of friction, energy converts between kinetic and potential.

\[
a_x = a_{\text{max}} \\
v_x = 0
\]

\[
a_x = \frac{1}{2} a_{\text{max}} \\
v_x = \pm \sqrt{\frac{3}{4}} v_{\text{max}}
\]

\[
a_x = \pm v_{\text{max}}
\]

\[
a_x = -\frac{1}{2} a_{\text{max}} \\
v_x = \pm \sqrt{\frac{3}{4}} v_{\text{max}}
\]

\[
a_x = -a_{\text{max}} \\
v_x = 0
\]

\[E = \begin{cases} U & \text{E is all potential energy.} \\ K & \text{E is partly potential, partly kinetic energy.} \end{cases}\]

\[E = \begin{cases} U & \text{E is all potential energy.} \\ K & \text{E is partly potential, partly kinetic energy.} \end{cases}\]
A Problem Using an Air Track – Example 11.5

- Refer to the solved problem on page 334.

(a) \[ F = 6.0 \text{ N} \]

(b) \[ m = 0.50 \text{ kg} \]

\[ x = 0 \quad x = 0.030 \text{ m} \]

\[ x = 0 \quad x = 0.040 \text{ m} \]
Reference Circle and Equations of Motion – Figure 11.20

• Place a salt shaker on a "lazy susan" and shine a bright flashlight from the side as it goes around, casting a shadow on a wall behind.

• Observe SHM in the motion of the shadow as the salt shaker traces out the "reference circle."

(a) Apparatus for creating the reference circle

(b) An abstract representation of the motion in (a)
As you follow the text on page 337, we obtain equations for velocity and acceleration.

While we can certainly reason about behavior at the midpoint or classical turning points, the explicit equations come from calculus.

We will state them as "given."
Graphs from a particle undergoing simple harmonic motion.

(a) Position as a function of time

(b) Velocity as a function of time

(c) Acceleration as a function of time
• Thinking about the physical properties of the spring and the mass of the glider.

• Refer to Example 11.6.
A Pendulum Undergoes Harmonic Motion – Figure 11.25

• The pendulum is a good example of harmonic motion.

• Oscillations depend on the length of the pendulum and the gravitational restoring.

The pendulum is a good example of harmonic motion.
Oscillations Can Be Damped – Figures 11.26 and 11.27

Upper cylinder attached to frame of car: remains relatively stationary

Lower cylinder attached to axle and wheel: moves up and down

Oscillator with strong damping force

Oscillator with weaker damping force

© 2016 Pearson Education, Inc.
Oscillations Can Be Driven – Figure 11.28

Each curve shows the amplitude for an oscillator subjected to a driving force at various angular frequencies.

Successive curves from blue to gold represent successively greater damping forces.

Lightly damped oscillators exhibit a sharp resonance peak when $\omega_d$ is close to $\omega$.

Strong damping reduces or eliminates the resonant peak.

Driving frequency $\omega_d$ equals angular frequency $\omega$ of oscillator.